Title: Positivity, negativity, entanglement, and holography

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Abstract: TBA
Positivity, Negativity, Entanglement, & Holography

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Entanglement ↔ Holography
- does the entanglement have to be EPR type?
Positivity, Negativity, Entanglement, & Holography
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Entanglement ↔ Holography
- does the entanglement have to be EPR-type?
- if entanglement were undistillable?
- classical correlations contamination?
- What kind of holographic constraints are to be imposed on QE?
- Matt's Q: what is special about HEE?
- What kind of holographic constraints are to be imposed on QE?

EE:
- Matt's Q: what is special about HEE?
- holographic entropy cone?
- are there alternate measures of QE which provide some intuition of how the holographic map work?
What kind of holographic constraints are to be imposed on QE?

\[
\{ \begin{align*}
EE & \quad \text{Matt's Q: what is special about HEE?} \\
& \quad \text{holographic entropy cone?} \\
& \quad \text{are there alternate measures of QE which provide some intuition of how the holographic map works?}
\end{align*} \]

Should every QI notion of entanglement witness have a geometric avatar?
Try to distinguish classical correlations from pure QE using entanglement negativity.

Bipartite system $\mathcal{H}_A \otimes \mathcal{H}_B$

density matrix $\rho$, $S_{A_{i_{\text{in}}}, B_{j_{\text{in}}}} = \langle \psi_{i_{\text{in}} j_{\text{in}}} | \rho | \psi_{i_{\text{in}} j_{\text{in}}} \rangle$
Try to distinguish classical correlations from pure QE using entanglement negativity.

Bipartite system $H_L \otimes H_R$

density matrix $\rho$, $\rho_{ab} = \langle r_a l_b | \rho | r_b l_a \rangle$

define partial transpose $\rho^T$, $\rho_{ab}^T = \rho_{ba}$
Given $g$, we define 2 measures of entanglement:

$$E(g) = \log ||g||_1$$

log negativity

The operation $\gamma$ is useful to talk about separability of density matrices.

$g$ is separable if

$$g = \sum\phi_i g_i$$

$\sum\phi_i = 1$

$\phi_i \geq 0$
Separable system $\mathcal{H}_1 \otimes \mathcal{H}_2$

Density matrix $\rho$

$S_{\text{max}} = \max_{S_{\text{max}}} S_{\text{max}}$

$S_{\text{max}} = S_{\text{max}}$

Given $\rho$, we define 2 measures of entanglement

Negativity $\mathcal{N} = \frac{1}{2} \log \left( \frac{\det \rho}{\det S_{\text{max}}} \right)$

by negativity $\mathcal{N}$ is useful to talk about separability of density matrices

$\rho$ is separable if $\mathcal{N} = 0$

$\rho$ is PPT $\implies \rho$ is not separable

$\rho$ is unentangled

$\rho$ has no negative eigenvalues $\rho$ is PPT $\implies \rho$ is not separable

$\rho$ is unentangled
\[ S \text{ is separable } \Rightarrow S^r \text{ has no negative eigenvalues} \]

\[ (S \text{ is PPT}) \]

\[ S \text{ is PPT } \Rightarrow S \text{ is undistillable} \]

\[ \mathcal{K}(S) \text{ gives a measure of the robustness of entanglement} \]

consider \( S \) & add noise \( S_s \sim \text{separable} \)

\[ \tilde{S} \sim \frac{1}{1+s} (S + sS_s) \]

\[ \text{measure of amount of noise} \]

Robustness \( \sim \min_{S \neq S_s} s = 2\mathcal{K}(S) \)
entanglement

Robustness ~ \( \min_{\hat{\rho}} S = 2N(\rho) \)

\[ \text{Tr} \left( \hat{\rho}^0 \right) \]

separability

\[ \Sigma \hat{\rho}_i = 1 \quad \hat{\rho}_i \geq 0 \]

Given \( S \) (\( H_{\text{L}} \oplus H_{\text{R}} \)), we can also consider \( \text{Tr}_L S = S_R \)

\[ \frac{N(S)}{S_R} \quad \frac{\Delta}{S_R} \]
Robustness \sim \min_{s} S = 2N(g)

\frac{\eta(g)}{\tilde{S}_K} \leq \frac{\xi}{S_z}

1. Compute \xi for a pure state

\left| \psi_{HH} \right> = \frac{1}{\sqrt{Z(p)}} \sum e^{-\beta E_a/2} \left| l_a, l_a > \right.

s = \left| \psi_{HH} \right> \left< \psi_{HH} \right|

\gamma(g) = \log \left( \frac{Z(p)_{e^g}}{Z(p)} \right)

\beta \left[ F(p) - F(p/2) \right]
Should every QI notion of entanglement witness have a geometric avatar?

For pure $\mathcal{S}$, $\mathcal{E}(\mathcal{S}) = S^0(\mathcal{S}_R)$

In 1+1 CFTs, use replica trick, w/ 1-change from usual EE/Renyi computation.

$s^5$ involves 1-swap of twist/anti-twist ds.
Should every QI notion of entanglement witness have a geometric avatar?

For pure $\mathcal{S}$, $\mathcal{E}(\mathcal{S}) = S^0(\mathcal{S}_t)$.

In 1+1 CFTs use replica trick, w/ 1 change from usual EE/Renyi computation.

$\mathcal{S}_t$ involves 1 swap of twist/anti-twist fields.

CFT$_2$, 1 interval:

$\mathcal{E}(10 \times 10) = \frac{\mathcal{E}}{2} \log \frac{M}{\epsilon}$
free theories \rightarrow \text{holographic theories}

\begin{align*}
\chi_{(3)} & \rightarrow 27 \\
\chi_{d=4} & \rightarrow 1.708 \rightarrow 0.98 \chi_{d=4} \text{ free}
\end{align*}

It is useful to talk about separability of density matrices. Is it separable?
twist \( \psi \)

\[ \begin{align*}
\text{free theories} & \rightarrow \text{holographic theories} \\
\chi_{(3)} & \rightarrow 27 \rightarrow 0.601 \chi_{\text{free}} \\
\chi_{\Delta=1} & \rightarrow 1.708 \rightarrow 0.98 \chi_{\Delta=4 \text{ free}} \\
\text{These were for } |0 \rangle_{cft} \leq 0 |1 \rangle \text{ partitioned across a spherical ball.}
\end{align*} \]
Given \( S \) \((H_\theta \otimes H_R)\) we can also consider \( \text{Tr}_S \) \( S = S_R \)

\[
\frac{\mathcal{U}(\theta)}{S_{IR}} \quad \frac{\Xi}{S_R}
\]

For \( \mathcal{D} A_g \)

\[
S^{(c)} = f_{\alpha}(q) R_{2A} + f_{\beta}(q) K_{2A} + f_{\gamma}(q) C_{2A}
\]

\( f_{\alpha}, f_{\beta}, f_{\gamma} \) are \( f \)'s of \( \theta \)

Renyi index & the central charge \((\theta, \gamma)\)
Given $e \in (H \otimes H)$, we can also consider $Tr e = 0$.

For $DA_g$ can be obtained from $DA = S$.

$S^{(a)} = f_a(q) R_{DA} + f_b(q) K_{DA}$

$S^{(b)} = f_c(q) C_{DA}$

$f_a, f_b, f_c$ are functions of $q$.

Rényi index $q$ and the central charges $(a, c)$.

$\chi = \frac{\mu_{univ}}{S_{univ}}$ can be less than unity if $DA$ is of sufficiently high genus in theories with $a > c$. 

$a = 1$ 

$DA_g$ is in the sector of $\mu_{univ}$.

$S_{univ}$ is the universal sector.
provide some intuition of how the holographic map work

Should every QI notion of entanglement witness have a geometric avatar?

Qubit Experiments $\mathcal{E}(\rho) = S^{\rho}$

pure states of a few qubits
Qubit Experiments

pure states of few qubits

\[ \sum_{2/12} N \quad \sum \quad \sum \quad \sum \quad \sum \quad \sum \]

when is monogamy of mutual information respected?

\[ I_3 = S_A + S_B + S_C - S_{AB} - S_{BC} - S_{CA} + S_{ABC} \]

\[ I_3 \leq 0 \]
- Under what conditions is monogamy of $S^2$ respected?

\[ S^2_{AIC} + S^2_{AIB} \leq S^2_{AIBC} \]

Holographically one finds situations where Araki–Lieb is saturated

\[ |S_A - S_{A'}} \leq S_{AUA'} \]

happens for states w/ low robustness and large mult-particle entanglement
Entanglement $\leftrightarrow$ Holography

- does the entanglement have to be EPR type?
- if entanglement were undistillable?
- classical correlations contamination?
- multipartite entanglement?
under what conditions is monogamy of $JS$ respected?

$JS_{AB} + JS_{AC} \leq JS_{ABC}$

holographically one finds situations where $\operatorname{Araki-Lieb}$ is saturated $|S_A - S_B| \leq S_{AB}$

$\text{MMI} \neq 0$ happens for states of low robustness and large multi-port entanglement