Particle-hole symmetry and the nature of the composite fermion

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DTS, PRX 5, 301027 (2015)
Plan

- Composite fermions in FQHE
- Problem of particle-hole symmetry
- Composite fermions as Dirac fermions
- Consequences
Composite fermions

Jain 1989

- CF = electron + even number of flux quanta
  - number of CFs = number of electrons
  - live in a reduced average magnetic field
- provide a unified explanations for a large number of QH plateaux
  - FQH of electrons ~ IQH of CFs
- $\nu=1/2$ state: a Fermi liquid of CFs
  - strong experimental evidence
HLR field theory

\[ \mathcal{L} = i\psi^\dagger (\partial_0 - iA_0 + i\alpha_0)\psi - \frac{1}{2m} |(\partial_i - iA_i + i\alpha_i)\psi|^2 + \frac{1}{2} \frac{1}{4\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \]

\[ b = \nabla \times a = 2 \times 2\pi \psi^\dagger \psi \]

“flux attachment”

mean field: \[ B_{\text{eff}} = B - b = B - 4\pi n \]

\[ \nu_{\text{CF}}^{-1} = \nu^{-1} - 2 \]

Halperin, Lee, Read 1993
Loose ends of CS field theory

Sometimes I feel a vague uneasiness with the flux-attachment procedure
Loose ends of CS field theory

Sometimes I feel a vague uneasiness with the flux-attachment procedure

D. Arovas

Flux $4\pi$ should do nothing, but does something
More loose ends

- Most common criticism of the HLR theory: lack of the explicit lowest Landau level projection

- Effective mass of CF finite while electron mass = 0, but in the vanilla HLR theory they are equal

- Can be fixed phenomenologically. Or simply accepted as an input in low-energy effective theory (Landau parameters).

- Perhaps the more fundamental problem is is the lack of particle-hole symmetry
**Particle-hole symmetry**

\[ \Theta |\text{empty}\rangle = |\text{full}\rangle \]

\[ \Theta c_k^+ \Theta^{-1} = c_k \]

\[ \Theta i \Theta^{-1} = -i \]

\[ \nu \rightarrow 1 - \nu \]

exact symmetry the Hamiltonian on the LLL, when mixing of higher LLs negligible

Girvin 1984
PH symmetry in the CF theory

PH conjugate pairs of FQH states

\[ \nu = \frac{n}{2n + 1} \quad \quad \quad \quad \nu = \frac{n + 1}{2n + 1} \]

\[ v=\frac{3}{7} \quad \quad \quad \quad \quad v=\frac{4}{7} \]

CF picture does not respect PH symmetry
PH symmetry of CF Fermi liquid?
Anti-Pfaffian state

- The Pfaffian state = p+ip BCS paired state of CFs
- What about the anti-Pfaffian state (particle-hole conjugate of Pfaffian)?
- Can it be p-ip BCS paired state of CFs?
  - No: the shift would be wrong (1 instead of -1)
  - can the anti-Pfaffian be
PH symmetry within HLR theory

- at $\nu=1/2$: PH symmetry requires $\sigma_{xy}=1/2$
- HLR + mean field: $\rho_{xy}=2$  $\sigma_{xy}<1/2$ Barkeshli Fisher Mulligan 2015
- Kivelson et al (1997): $\sigma_{xy}=1/2$ requires CFs to have nonzero Hall conductivity at zero field

$$\sigma_{xy}^{\text{CF}} = -\frac{1}{2} \frac{e^2}{h}$$

while within HLR:  $\sigma_{xy}^{\text{CF}} = 0$

A new view on the CFs

- CF is a Dirac fermion: Berry phase $\pi$ around Fermi surface
- CF transforms to CF (not hole) under PH conjugation
- CFs interact through an emergent U(1) gauge field, whose action does not have the Chern-Simons term
- The number of CFs is half the magnetic flux, in general not equal to the number of electrons

DTS, PRX 5, 301027 (2015)
Wang, Senthil
Metlitski, Vishwanath; Metlitski
Geraedts et al., 1508.04140
Mross, Alicea, Motrunich, 1510.08455
...

First indication of Dirac nature of CFs

\[ \nu = \frac{n}{2n + 1} \quad \rightarrow \quad \nu_{CF} = n \]

\[ \nu = \frac{n + 1}{2n + 1} \quad \rightarrow \quad \nu_{CF} = n + 1 \]
First indication of Dirac nature of CFs

\[ \nu = \frac{n}{2n+1} \]

\[ \nu_{\text{CF}} = n + \frac{1}{2} \quad ? \]

CFs form an IQH state at half-integer filling factor: must be a Dirac fermion
IQHE in graphene

\[ \sigma_{xy} = \left( n + \frac{1}{2} \right) \frac{e^2}{2\pi\hbar} \]

**Figure 4: QHE for massless Dirac fermions.** Hall conductivity \( \sigma_{xy} \) and longitudinal resistivity \( \rho_{xx} \) of graphene as a function of their concentration at \( B = 14 \, \text{T} \) and \( T = 4 \, \text{K} \). \( \sigma_{xy} = (4e^2/h)\nu \) is calculated from the measured
Origin of anomalous Hall conductivity of the CFs

- PH symmetry requires the CFs to have nonzero anomalous Hall conductivity equal $-1/2 \, e^2/h$ (Kivelson et al 1997)

- Haldane and others: fractional part of anomalous Hall conductivity = Berry phase around Fermi surface

- Thus the Berry phase of the CFs have to be $\pi$
**PH conjugation**

\[ \Theta |\text{empty}\rangle = |\text{full}\rangle = c_1^\dagger c_2^\dagger \cdots c_M^\dagger |\text{empty}\rangle \quad \Theta c_k^\dagger \Theta^{-1} = c_k \]

\[ \Theta^2 |\text{empty}\rangle = (-1)^{M(M-1)/2} |\text{empty}\rangle \]

\[ \Theta^2 |\text{any}\rangle = (-1)^{M(M-1)/2} |\text{any}\rangle \quad |\text{any}\rangle = c_{n_1}^\dagger c_{n_2}^\dagger \cdots c_{n_{N_e}}^\dagger |\text{empty}\rangle \]

\[ M = 2N_{CF} \quad \Theta^2 = (-1)^{N_{CF}} \]

For consistent \( \Theta^2 \) assignment, number of CFs should not be the number of electrons

Geraedts, Zaletel, Mong, Metlitski, Vishwanath, Motrunich; Levin, Son
Dirac composite fermions

- The notion of a Dirac composite fermion was proposed by Mross, Essin, Alicea (2014) for the interacting surface of TI

- In the context of QHE, it may seem strange: the original electron is nonrelativistic

- But in the lowest Landau level limit there is no difference between nonrelativistic or relativistic electrons

\[ \nu_{\text{rel}} = \nu_{\text{NR}} - \frac{1}{2} \]
Flux attachment?

- Since number of CFs ≠ number of electrons, flux attachment is not the right picture
- What is the alternative?
Jain sequences

In relativistic convention \( \nu_{\text{rel}} = \nu_{\text{NR}} - \frac{1}{2} \)

Jain's sequences \( \nu_{\text{NR}} = \frac{n}{2n + 1} \) and \( \nu_{\text{NR}} = \frac{n + 1}{2n + 1} \)

correspond to \( \nu_{\text{rel}} = \pm \frac{1}{2(2n + 1)} \)

We want \( \nu_{\text{CF}} = n + \frac{1}{2} \)

\[ 2\nu_{\text{rel}} = \frac{1}{2\nu_{\text{CF}}} \]
Particle-vortex duality

- If electron-electron interaction is short-ranged, the CF theory is QED3: non-Fermi liquid
- For Coulomb interactions: marginal Fermi liquid
- Also useful for understanding strongly interacting surface of TI (Metlitski’s talk)
EM response

- Electron conductivity tensor $\sigma$ in terms of the CF conductivity tensor $\tilde{\sigma}$

\[
\sigma_{xx} = \frac{1}{4} \frac{\tilde{\sigma}_{xx}}{\tilde{\sigma}_{xx}^2 + \tilde{\sigma}_{xy}^2},
\]

\[
\sigma_{xy} = -\frac{1}{4} \frac{\tilde{\sigma}_{xy}}{\tilde{\sigma}_{xx}^2 + \tilde{\sigma}_{xy}^2} + \left[\frac{1}{2}\right]_{NR}.
\]

(all in units of $e^2/h$)
Galilean invariance and Kohn's theorem

\[ \mathcal{L} = i \psi^\dagger (D_0 + v_F \mathbf{\sigma} \cdot \mathbf{D}) \psi + \frac{i}{2} \mathbf{v} \cdot \psi^\dagger \mathbf{D} \psi - \frac{1}{4} (\nabla \times \mathbf{v}) \psi^\dagger \mathbf{\sigma}_3 \psi + \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda - \frac{b}{8\pi} (\nabla \times \mathbf{v}) \]

where \( D_\mu \psi = (\partial_\mu + 2ia_\mu) \psi \).

relaxation time approximation

\[
\sigma_{xx} = \frac{1}{4} \frac{\tilde{\sigma}_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} + \frac{m_*}{2B} i\omega ,
\]

\[
\sigma_{xy} = -\frac{1}{4} \frac{\tilde{\sigma}_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} + [\frac{1}{2}]_{\text{NR}} .
\]

\[
\sigma_{xx} = \frac{m_*}{2B\tau} ,
\]

\[
\sigma_{xy} = \frac{b}{B} + \frac{1}{2} = \nu .
\]
\[ \vec{v} = \frac{\vec{E} \times \hat{z}}{B} \]
Consequences

• Exact particle hole symmetry in linear response
  • at $\nu = \frac{1}{2}$, $\sigma_{xy} = \frac{1}{2}$ exactly (HLR: $\rho_{xy}=2$)

• New particle-hole symmetric gapped nonabelian state at $\nu=1/2$ “PH-Pfaffian”
  $$\langle e^{\alpha\beta} \psi_\alpha \psi_\beta \rangle \neq 0$$

  Quantum Hall version of the T-Pfaffian in TI

Pfaffian and anti-Pfaffian states: pairing of Dirac CFs with angular momentum 2 and -2
HLR theory as the NR limit

When CP is broken, CF has mass
In the NR limit: NR action for CF
Integrating out Dirac sea: Chern-Simons interaction between CF

Old HLR theory is reproduced
Particle-hole symmetry broken by the CF Dirac mass
Numerical evidence of Dirac CFs

- Suppression of back-scattering on particle-hole symmetric defect

- Observed

Geraedts, Zaletel, Mong, Metlitski, Vishwanath, Motrunich
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Wang-Senthil picture of the CF

Wang, Sentil 1507.08290
Conclusion and open questions

- PH symmetry: a challenge for old CF picture
- Proposal: Dirac CF with gauge, non-CS interaction
- particle-vortex duality instead of flux attachment
- Open questions:
  - a better derivation of the effective theory (but see Mross, Alicea, Motrunich 1010....)
  - experimental measurement of the Berry phase?
  - other experimental signatures?