Abstract: Formal loop spaces are algebraic analogs to smooth loops. They were introduced and studied extensively in the 2000' by Kapranov and Vasserot for their link to chiral algebras. In this talk, we will introduced higher dimensional analogs of K. and V. formal loop spaces. We will show how derived methods allow such a definition. We will then study their tangent complexes: even though formal loop spaces are "of infinite dimension", their tangent has enough structure so that we can speak of symplectic forms on them.
\( d = 1 \) \((KV)\) \( L^1_v(X) = \mathcal{Y}_p(\hat{\Lambda}, X) \) \( \lambda \rightarrow \)
Formal loop spaces:

$d = \pm (KV) \quad L_n'(X) = \pi_{KP}(\hat{A}', X) : A \to X(A^{[n]})$

$L_0'(X) = \pi_{KP}(\hat{A}'|_{so(1)}, X) : A \to X(A^{[0]})$

we impose $X \to L_n'X$ be co-descent.

$L_n'X$ = formal completion of $L_n'(X)$ into $L_0'(X)$. 
Formal loop spaces:

d=1: (KV): \[ L_v'(X) = \pi_0(\hat{A'}(X), A) \rightarrow X(A[v]) \]

\[ L_u'(X) = \pi_0(\hat{A'}(X), A) \rightarrow X(A_{(1)}) \]

\[ X \rightarrow L'X \text{ is a codescent.} \]

\[ L'X = \text{formal completion of } L_v'(X) \text{ into } L_u'(X). \]
Formal loop spaces:

\[ L_\nu (X) \approx \text{Hom}(\hat{A}'', \mathbb{P}^1_\nu \to X) : A \mapsto X(A \llcorner (1)) \]

\[ L_\nu (X) = \text{Hom}(\hat{A}' \to \mathbb{P}^1_\nu \to X) : A \mapsto X(A \llcorner (1)) \]

Then (KV):
- \( X \twoheadrightarrow \mathcal{L}' X \) has \( \mathcal{L}' X \) is representable by an ind-scheme.
- \( \mathcal{L}' X \) is \( A \mapsto \mathcal{L}' X \) as \( \mathcal{L}' X \) is a co-descent.

\[ \mathcal{L}' X = \text{formal completion of } \mathcal{L}' \mathcal{X} \text{ into } \mathcal{L}' X \]
\[
\begin{align*}
L^d_\nu(X) &= \mathcal{L}_\nu(X) \otimes \mathbb{Q}, \\
\mathcal{L}_\nu(X) &= \pi_2^G(\hat{\mathbb{A}}^X, X), \quad A \rightarrow X(\mathbb{A}^{\text{ct}}, \nu : A)
\end{align*}
\]

\[
\begin{align*}
d \geq 1 : \\
\mathcal{L}_\nu(X) &= \pi_2^G(\hat{\mathbb{A}}^X, X), \quad A \rightarrow \mathbb{R} \langle \text{Spec}(\mathbb{A}^{\text{ct}}, \nu : A) \rangle \setminus \{X\}, X
\end{align*}
\]
\[ \mathcal{L}^d(X) = \text{frob completion of } \mathcal{L}^d(X) \text{ into } \mathcal{L}^d_0(X). \]

\[ \begin{align*}
\mathcal{L}^d(X) & = \mathcal{L}^d_{\text{frob}}(X) \\
& = \mathcal{L}^d_{\text{frob}}(\mathcal{C}^d \{ \text{Spec} \mathcal{O}_X \} \setminus \{ \}, X)
\end{align*} \]

Ex. \( X = \mathbb{A}^1 \), pb: \( \mathcal{L}^d_{\text{frob}}(\mathbb{A}^1) \leq \mathcal{L}^d_{\text{frob}}(\mathbb{A}^1) \) for \( d \geq 2 \).

\[ \text{VI DAG}. \]
\[ L^d X = \text{formal completion of } L^d Y \text{ into } L^d X \]

\[ d \geq 1. \quad \text{Def: } L^d V (X) = \tau_0 (\mathbb{A}^d, X) : A \to X(\mathbb{A}^d, t_{A, d}) \]

\[ L^d V (X) = \tau_0 (\mathbb{A}^d \setminus \{ 0 \}, X) : A \to \tau_0 (\text{Spec} (\mathbb{A}^d \setminus \{ 0 \}), X) \]

Ex: \( X = \mathbb{A}^1 \) \( \implies \) \( L^d V (\mathbb{A}^1) \leq L^d V (\mathbb{A}^1) \) \( \forall d \geq 2 \) if not closed

Def: \( L^d X = \text{formal neighborhood of } L^d Y \text{ into } L^d X \)
Thm (-):

- $X \rightarrow Y$ satisfies étale descent.
  (even smooth descent if $\pi = 1$)

- $Y^{-} \leftarrow X$ is represented by a derived ind-pro scheme.
Proof: Admit $X = \text{Spec}A$.

To prove $\omega$. 
I'm not sure what the content of the board is. It seems like there might be some mathematical notation or a description of a proof. Can you provide a clearer image or description of the content on the board?
Proof. Admit 0.

To prove \( \varnothing \) no descent as \( X = \text{Spec} A \).

generators and relations - \( X \text{-lim } \frac{A'}{A'} \).

\( \frac{A'}{A} = \bigcup_\varnothing \frac{A'}{A} \text{ rel } \frac{A'}{A} \).

Reduce to \( X = \frac{A'}{A} \).
Proof: Admit \( D \).

To prove \( \mathcal{O} \) no descent is \( X = \text{Spec} A \).

Generators and relations is \( X = \text{lim} \, \Lambda' \).

\( \text{no reduce to } X = \Lambda' \).

\[ \Lambda[x,y] = \text{colim} \Lambda[x,y] \]

\[ \text{Spec} (\Lambda[g, x, y; t'_{1}, t'_{2}]) \]

ind proj diagram
Proof. Admit \( \Omega \). To prove \( \otimes \) no descends \( X = \text{Spec} A \). generator and relations \( X = \lim A' \). Reduce to \( X = A' \).

At the end:

\[
\text{Spec}(A^e, A^e(2))[E^{-1}] \xrightarrow{\text{indep. diagram}} \text{Spec}(k[t_1, \ldots, t_n])[E^{-1}]
\]
Theorem (\text{-})

$X \to X^{et}$ satisfies étale descent.

(even smooth descent, $p = 1$).

$Y^{d, X}$ is represented by a derived ind-pro-scheme of $Y$. 

$H^n(\text{Coh}(Y^{d, X})) = 0$ if $n = 0$.

$H^n(\text{Coh}(Y^{d, X})) = 0$ if $n = d - 1$. 

\[ H^n(\text{Coh}(Y^{d, X})) = 0 \text{ if } n = 0, \] 

\[ H^n(\text{Coh}(Y^{d, X})) = 0 \text{ if } n = d - 1. \]
Thm (-):

1. $X \rightarrow X^d$ satisfies étale descent.
   (even smooth descent if $d=1$).

2. $X^d$ is represented by an ind-pro-scheme.

\[ H^n(A', S_0) = \begin{cases} \mathbb{L}(t-d) & \text{if } n=0 \\ (t-d)^{-n} H(A, t) & \text{if } n=d-1 \\ 0 & \text{else} \end{cases} \]
Vector spaces

(Lefschetz, Tate, Beilinson,
Drinfeld, Berning,
Grothendieck,
Wald.)

\( V \) top vector space.

Tate twist \( V = W \oplus W' \)
direct dual of a discrete.
Vector spaces
(Lefschetz, Tate, Beilinson, Deligne, Brauer, Grothendieck, Wolfson)

*V* is a vector space.

Tate vector: \( V = W_0 \oplus W \)

discr = dual of a discrete
Vector spaces

(Lefschetz, Tate, Bedlinon
Dunfield, Brahmity-
Göcking-
Wolff)

$
\mathcal{V} \text{ top vec space}
$

Tate work

$V = W_1 \oplus W_2$

direct

dual of a discrete.

Proof

Final Proof

Proof

$\mathcal{V}_1 \rightarrow \mathcal{V}_2 \rightarrow \mathcal{V}_3$
Def. $\mathcal{E}$ st. and id. comp $(\mathcal{E},1)$-act.

$$\text{Tate}(\mathcal{E}) \rightarrow \text{ProInd}(\mathcal{E})$$

Smallest full subcat st. stable, id. comp.

and contains $\text{Ind}\mathcal{E}$ and $\text{Pro}\mathcal{E}$.

Def. $L^d X = \text{funal neighborhood of } L^d X \text{ into } X.$
Def. $C$ st and id. comp. $(\otimes, l)$-af.

Tate$(C) \rightarrow$ ProInd$(C)$

Smallest full subcat st stable, id. comp.
and contains Ind$C$ and Pro$C$.

Thm:

1. Tate$(C)$ has a uniq. prop \( \Rightarrow \)
   Tate$(C) \rightarrow$ IndPro$(C)$

2. $K^m$ (Tate$C) \in \subseteq K^m(C)$
Def: $E$ st and id. comp. $(\alpha, 1)$-af

Tate$(E) \rightarrow$ ProInd$(E)$

Smallest full subcat st stable, id. comp.

and contains Ind$E$ and Pro$E$.

Thm:

1. Tate$(E)$ has a uniq. prop as
   Tate$(E) \rightarrow$ IndPro$(E)$

2. $K^c_n$(Tate$E) = \Sigma K^c_n(E)$
Theorem: 

\[ \text{If } L^d x \text{ and } L^d y \text{ are Tate modules over } \mathbb{Z}_p x \]

Proof: Reduce to \( X = H^d \).

\[ L^d H^d = \text{cyclic} \quad \text{and} \quad \text{d}(p^m) \]
Theorem: $\tilde{\mathcal{O}}_{\mathcal{O}}(\mathcal{O})$ and $\tilde{\mathcal{O}}_{\mathcal{O}}(\mathcal{O})$ are Tate modules over $\mathcal{O}$.

Proof: Reduce to $\mathcal{O} = \mathbb{A}^1$.

$\tilde{\mathcal{O}}_{\mathcal{O}}(\mathcal{O})$ is cotangent complex $L^1_{\mathcal{O}/\mathcal{O}} = \text{colim}_{x \in \mathcal{O}} x^{\infty}$. If $\text{char}(k) = p$, then $\tilde{\mathcal{O}}_{\mathcal{O}}(\mathcal{O}) = \text{colim}_{x \in \mathcal{O}} x^{\infty}$. If $\text{char}(k) = 0$, then $\tilde{\mathcal{O}}_{\mathcal{O}}(\mathcal{O}) = \text{colim}_{x \in \mathcal{O}} x^{\infty}$.
Joint with F. Karpil. 

Idea: $d \geq 2$

$\mathbb{L}^d(Ba) \rightarrow K(x^d)_{a},Z$

Def: $\mathbb{L}^d x = \text{final neighborhood of } \mathbb{L}^d x \text{ into } \mathbb{L}^d x$. 
KV \mapsto g((t)) \rightarrow \mathfrak{h} C^1 \text{ map into class in } C^2(g((t))).

\text{Koszul}: C^\infty(\text{End}(g((t))))

\text{End } (g((t))) \rightarrow \mathfrak{h} C^1

Trace trace.

d \geq 2:

L^d(BG) \rightarrow \mathfrak{h} C^1
\[
\text{We have such a class.}
\]