Title: A perspective on derived analytic geometry

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Abstract: I will present a 'categorical' way of doing analytic geometry in which analytic geometry is seen as a precise analogue of algebraic geometry. Our approach works for both complex analytic geometry and p-adic analytic geometry in a uniform way. I will focus on the idea of an 'open set' as used in various geometrical theories and how it is characterized categorically. In order to do this, we need to study algebras and their modules in the category of Banach spaces. The categorical characterization that we need uses homological algebra in these 'quasi-abelian' categories which is work of Schneiders and Prosmans. In fact, we work with the larger category of Ind-Banach spaces for reasons I will explain. This gives us a way to establish foundations of derived analytic geometry (my joint project with Kobi Kremnizer). We compare this approach with standard standard notions such as the theory of affinoid algebras, Grosse-Klonne's theory of dagger algebras (over-convergent functions), the theory of Stein domains and others. I will formulate derived analytic geometry following the relative algebraic geometry approach of Toen, Vaquie and Vezzosi.

This talk involves various joint work with Federico Bambozzi and Kobi Kremnizer.
A perspective on derived analytic geometry

joint w/ Kremnizer
idea of using relative algebraic geometry
A perspective on derived analytic geometry

joint w/ Kremnizer
idea of using relative algebraic geometry

Under Spec \( \mathbb{Z} \) (Toën-Vaquie)
HAG (Toën-Vezzosi)
HAG develops derived geometry w/ input a monoidal model category

HAG context

\[ \text{ex} \cdot \text{Ab} \]

\[ - \cdot \text{ch} \leq_0 \quad \text{char k} = 0 \]

\[ \ast \cdot \text{sAb} \]
\( \exists - A_b \)

\( - c_{h k} \leq 0 \)

\( \text{char} k = 0 \)

\( \leq s A_b \)

suggestion: \( M = s \text{Ind}(B a n R) \)

Thm \((R, \text{Krennizer})\) \( M \) is a HAG context

\( R \) a Banach ring
A Banach ring is a ring $R$ equipped with $R ightarrow R_{\geq 0}$ such that:

- $|a+b| \leq |a| + |b|$
- $|ab| \leq C|a||b|$
- $|a| = 0 \Rightarrow a = 0$

$d(a,b) = |a-b|$ is complete

$R = \mathbb{Z}, \mathbb{C}, \mathbb{R}, \mathbb{Q}_p, \mathbb{CC}(t), \mathbb{Non}$
\[ \text{Ban}_R \text{ has finite limits/collimits} \]

\[ \text{objects are } (V, \| \|) \]

\[ V \text{ an } R \text{-module} \]

\[ \| v + w \| \leq \| v \| + \| w \| \]

\[ \| av \| \leq C \| a \| \| v \| \]
\[ \text{Ban}_R \text{ has finite limits/cokimits, no infinite prods or coprods} \]

objects are \((V, \| \cdot \|)\)
morphisms are bounded

\(V\) an \(R\)-module

\[ \| v + w \| \leq \| v \| + \| w \| \]

\[ \| av \| \leq C \| a \| \| v \| \]

\[ d(v, w) = \| v - w \| \text{ complete} \]

\[ \| v \| = 0 \iff v = 0 \]
\[ \text{Ban}_R \text{ has finite limits/cokimits, no infinite prods or coprods} \]

objects are \((V, \| \cdot \|)\)

\(V\) an \(R\)-module

\[ \|v + w\| \leq \|v\| + \|w\| \]

\[ \|av\| \leq C \|a\| \|v\| \]

\[ d(v, w) = \|v - w\| \text{ complete} \]

\[ \|v\| = 0 \iff v = 0 \]
\[ d(v, w) = \|v - w\| \text{ complete} \]

\[ \|v\| = 0 \iff v = 0 \]

\[ \text{Ban}_R \text{ has all limits/colimits} \]

\[ \bigoplus_{i \in I} V_i \leq X V_i, \quad \| (v_i)_{i \in I} \| = \sum_{i \in I} \| v_i \| \]

\[ \sum_{i \in I} \| v_i \| < \infty \]
\[ d(v,w) = \|v-w\| \] complete
\[ \|v\| = 0 \iff v = 0 \]

Ban \( \mathbb{R} \)

\[ \bigoplus V_i = X \]
\[ \bigoplus \|v_i\| = \sum_{i \in I} \|v_i\| \]

\[ \|v\| < \infty \]
$d(v,w) = \|v-w\|$ complete

$\|v\| = 0 \iff v = 0$

$\Sigma_{i=1}^{n} \|v\|_i < \infty$

$B_{\|\cdot\|} \text{ is closed symmetric monoid}$
\[ d(v,w) = \|v-w\| \text{ complete} \]
\[ \|v\| = 0 \iff v = 0 \]

\[ \prod_{i \in I} v_i = x \prod_{i \in I} v_i \quad \| (v_i)_{i \in I} \| = \sum_{i \in I} \|v_i\| \leq A \]

\[ \sum_{i \in I} \|v_i\| < \infty \]

\[ \text{Ban}_R \text{ is closed symmetric monoid} \]

\[ V \otimes W \sim \text{completion of } V \otimes W \]

\[ w/v \quad \|w\| = \inf \left\{ \sum_{i=1}^n \|v_i\| : v_i \in \mathbb{R} \right\} \]
\[ \| \mathbf{V} \| = \sum_{i \in I} \| \mathbf{v}_i \|, \quad \| (\mathbf{v}_i)_{i \in I} \| = \sum_{i \in I} \| \mathbf{v}_i \| \]

\[ \sum_{i \in I} \| \mathbf{v}_i \| < \infty \]

Ban$_R$ is closed symmetric monoid

Ban$_R$ is not abelian

V @ W \leftarrow \text{completion of } \frac{V \otimes W}{R} \quad \text{w.r.t.} \quad \| \mathbf{u} \| = \inf \left\{ \sum_{i=1}^{n} \| \mathbf{v}_i \| \mid u = \sum_{i=1}^{n} \mathbf{v}_i \right\}
\[ \sum_{i=1}^{n} \| v_i \| < \infty \]

Ban R is closed symmetric monoidal

\[ \forall \omega \in \overline{V} \]

Ban R is not abelian

\[ f : V \to W \]

\[ \text{cok}(\ker f \to V) = \ker(h : W \to \text{cok} f) \]

\[ \| w \| = \inf \left\{ \sum_{i=1}^{n} \| w_i \| \right\} \]

May not hold
\( \bigoplus_i V_i \cong X \), \( \| (V_i)_{i \in I} \| = \sum_{i \in I} \| V_i \| \)

\( \sum \| V_i \| < \infty \)

\( \text{Ban}_R \) is closed symmetric monoidal

\( V \otimes W \leftarrow \text{completion of } V \otimes W \)

\( \| u \| = \inf \left\{ \sum_{i=1}^{n} \| u_i \| \mid u = \sum_{i=1}^{n} u_i \otimes w_i \right\} \)

\( \text{Ban}_R \) is not abelian

\( f : V \to W \) \( \text{cok}(\ker f) \to V = \ker(\text{cok} f) \) may not hold, \( f \) is strict if it holds
Ban \( R \) is quasiabelian — developed by J. P. Schneiders

\[ \forall f \in W, \text{ cok} f = \frac{W}{H} \]

not identifiable

\[ \forall \]
Ban\(_R\) is quasi-abelian — developed by J. P. Schneiders.

A object is flat if tensoring by it preserves strict s.e.s.

cof\(f = w\)

not identified
Ban \( A \) is quasi-abelian — developed by J.P. Schneiders

\[ V \xleftarrow{f} W \quad \operatorname{cod} f = W \]

\[ \text{not identified} \]

\[ \text{Def. A object is flat if tensoring by it preserves strict s.e.s.} \]

\[ P \text{ is projective if } \forall E \rightarrow F \text{ strict epi} \]

\[ \operatorname{Hom}(P, E) \rightarrow \operatorname{Hom}(P, F) \]
Ban$_R$ is quasi-abelian — developed by J. P. Schneiders

\[ \forall f \in W, \text{ cok} f = W_{/V} \]

**Def.** An object is flat if tensoring by it preserves strict s.e.s.

- $P$ is projective if $\forall E \rightarrow F$ strict epimorphism:
  \[ \text{Hom}(P, E) \rightarrow \text{Hom}(P, F) \]

\[ P_{/V} \text{ strictepi} \]

\[ P_{/V} \text{ flat proj} \]
$\text{Ind (Ban}_R\text{)}$ again quasi-abelian
Closed, Sym, monoidal
\[ \text{Ind} \left( \text{Ban}_{\mathcal{R}} \right) \hspace{1cm} \text{closed, symmetric monoidal} \]
\[ \big( \boxtimes_i \colim_j V_i \big) \boxtimes_j \colim_i W_j \]
\[ = \colim_i \left( V_i \boxtimes_j W_j \right) \]
\[ = \lim_j \left( \colim_i V_i \right) \boxtimes_j \colim_i W_j \]
Ind₂(\text{Ban}_R) \text{ again quasi-compact, closed, sym, monoidal}

\text{colim}_i \text{"} V \text{"}_i \otimes \text{colim}_j \text{"} W \text{"}_j

= \text{colim}_i \text{"} (V \otimes \text{"} W \text{"}_j)

\text{Hom}_i \text{"} (\text{colim}_i \text{"} V \text{"}_i, \text{colim}_j \text{"} W \text{"}_j) = \text{lim}_j \text{"} \text{Hom}_i \text{"} (V \text{"}_i, W \text{"}_j)

\text{C Born} \leq \text{Ind}_2 (\text{Ban}_R)

\text{Idea of using relative algebraic geometry}

\text{Under Spec} \mathbb{Z} \text{ (Toën-Vaquié)}

\text{HAG (Toën-Vezzosi)}
\[ \text{Ind}(\text{Ban}_R) \text{ again quasi-abelian} \]

\[ \text{closed, Sym, monoidal} \]

\[ \text{collin} \left( \text{colim} \left( V_i \right) \right) \neq \text{colim} \left( \text{colin} \left( W_j \right) \right) \]

\[ = \text{colini} \left( \text{colim} \left( V_i \right) \right) \otimes \text{colin} \left( W_j \right) \]

\[ \text{Hom} \left( \text{colin} \left( V_i \right), \text{colin} \left( W_j \right) \right) = \text{lim} \text{colin} \left( \text{Hom} \left( V_i, W_j \right) \right) \]

\[ \text{CBorn} \leq \text{Ind}(\text{Ban}_R) \]

\[ V = \text{Hom}_R \left( V, R \right) \]

\[ \text{Under Spec} \mathbb{Z} \left( \text{Toën-Vaquié} \right) \]

\[ \text{HAG} \left( \text{Toën-Vezzosi} \right) \]
Def \( V \in \text{Ind}(\mathbb{B}_{\text{Ban}}^R) \) is nuclear

if \( \mathcal{K} \in \mathbb{B}_{\text{Ban}}^R \)

\[
\text{Hom}_R(\mathcal{K}, V) = \mathcal{K}_R \otimes V
\]
Def \( V \in \text{Ind}(\text{Ban}_R) \) is nuclear
if \( \forall \psi \in \text{Ban}_R \)
\[
\text{Hom}_R(\psi, V) = \psi \otimes_R V
\]
\( S\text{Ind}(\text{Ban}_R) \) has model structure
\( X \to \mathbb{K} \) is a weak \( \mathcal{A} \mathcal{P} \) projective
Def \( V \in \text{Ind}(\text{Ban}_R) \) is nuclear if \( \forall w \in \text{Ban}_R \)

\[ \text{Hom}(w, V) \cong w^* \otimes V \]

\( S\text{Ind}(\text{Ban}_R) \) has model structure

\( X \rightarrow K \) is a we/fib \( \Rightarrow P \) projective

\( \text{Hom}(P, X) \rightarrow \text{Hom}(P, Y) \) is a we/fib
Def \( V \in \text{Ind}(\text{Ban}_R) \) is nuclear
if \( \not\exists \text{ We } \text{Ban}_R \)
\[
\text{Hom}(W, V) \cong W^\ast \otimes_R V
\]
\( \text{Ind}(\text{Ban}_R) \) has model structure
\( X \to K \) is a \( \text{we}/\text{Af} \) \( \Rightarrow \) \( \text{P projective} \)
\[
\text{Hom}(P, X) \to \text{Hom}(P, Y)
\]
Def $V \in \text{Ind}(\text{Ban}_R)$ is nuclear if for all $W \in \text{Ban}_R$
$$\text{Hom}(W, V) \cong W \otimes_R V$$

$\text{Ind}(\text{Ban}_R)$ has model structure.

$X \to K$ is a we/af $\Leftrightarrow$ $\text{P}$ projective.

$\text{Hom}(P, X) \to \text{Hom}(P, Y)$ is a we/af gives a model cat str.
\[ \text{Conn(Spec(Rng))} \]
\[ (X \otimes Y)_n = X_n \otimes Y_n \]
\[ \text{Affine Schemes} = \text{Con}(\text{sInd}(\text{Ban}_n)) \]
Affine Schones = Cons(s Ind(\text{Range}))^T
A \in \text{Comm}(s \text{ Ind}(\text{Range}))
\text{Mod}(A)
Affine Scheme = \text{Cons}(s, \text{Ind}(\text{End}(\mathcal{R}_2)))

A \in \text{Comm}(s, \text{Ind}(\text{End}(\mathcal{R}_2)))

\text{Mod}(A)

A \otimes M \to M
\( A \in \text{Comm}(s \text{Ind}(\text{Ban}_R)) \)

\( \text{Mod}(A) \)

\( \Rightarrow \text{model category} \)

\( \text{Formal Cariski Topology} \)

\( \text{on } \text{Comm}(s \text{Ind}(\text{Ban}_R)) \)
Formal Zariski Topology
on \( \text{Comm} \left( \text{Incl}(\text{Bar}_R) \right) \)

\[ \exists A \rightarrow B : \exists i \in I \text{ is a cover} \]
A ∈ \text{Comm}(\text{Ind}(\text{Ban}_2))
\text{Mod}(A)
A \otimes M \to M 
\text{model category}

\underline{\text{Formal Zariski Topology}}
\text{on } \text{Comm}(\text{Ind}(\text{Ban}_2))
\exists A \to B \equiv \exists i \in I \text{ is a cover}
\text{if } J \subseteq I \text{ finite}
Formal Zariski Topology
on $\text{Comm}(\text{CInf}(\text{BanR}))$

1. $A \to B_i$ for $i \in I$ is a cover
2. If $J \subseteq I$ finite st.
3. $\text{Ho}(\text{Mod}(B_i)) \to \text{Ho}(\text{Mod} A)$ is ff, $i \in J$
4. $M \to N$ in $\text{Ho}(\text{Mod} A)$ is isom
Formal Zariski Topology
on \( \text{Comm}(\mathfrak{S}_{\text{Ind}(\text{Ban}_R)}) \)

\[ \exists A \to B, \exists i : I \text{ is a cover} \]
if \( J \leq I \) finite set.

\[ \text{Ho}(\text{Mod}(B_i)) \to \text{Ho}(\text{Mod}(A)) \text{ is ff, } \forall i \in J \]

\[ M \to N \text{ in } \text{Ho}(\text{Mod}(A)) \text{ is isom} \]
\[ M \otimes B \to M \otimes B_i \text{ is isom } \forall i \in J \]
Formal Zariski Topology

on $\text{Comm}(\text{Incl}(\text{Ban}))$

$\exists A \rightarrow B_i \quad \exists i \in I$ is a cover if $J \leq I$ finite st.

1. $\text{Ho}(\text{Mod}(B_i)) \rightarrow \text{Ho}(\text{Mod}(A))$ is ff. $A \in J$

2. $M \rightarrow N$ in $\text{Ho}(\text{Mod}(A))$ is isom. $M$ & $\forall B_i \rightarrow M \otimes B_i$ is isom. $A \in J$
\[ M = sA_{\text{AB}} \]

(1) \[ B_i \otimes_{A} B_j \sim B \]
\[ M = sA B \]

(1) \[ B_i \otimes B_i \cong B_i \]

homotopy epi ← being a flat epi
$M = sA \underline{B}$

1. $B; \odot B; \sim B;
   \text{homotopy epi} \iff \text{being a flat epi}

$A \Rightarrow B$ is of f.p. then flat epi = ko epi

$A \Rightarrow A_p$
\[ M = sA \text{Ab} \]

(1) \( B_i \times B_i \Rightarrow B_i \) is a flat epi if \( A \rightarrow B_i \) is a flat epi

**Homotopy epi** ↔ being a flat epi

\[ A \rightarrow A \text{ flat} \Rightarrow \text{flat proj} \]
\[ M = sA \text{ and } B \otimes B = B \]

(1) \[ B : \frac{1}{\text{homotopy epi}} \Rightarrow B : \frac{1}{\text{being a flat epi}} \]

\[ A \Rightarrow B \text{ is of fp, then flat epi } = \text{ ho epi} \]

\[ \Rightarrow \text{Spec } A \text{ is flat, Zar open} \]

\[ A \Rightarrow \text{Spec } P \text{ is flat, proj} \]

\[ P \Rightarrow K \text{ is flat, proj} \]
\[ M = sAB \]

(1) \[ B; \otimes B; \simeq B; \]

\[ \text{homotopy epi} \leftrightarrow \text{being a flat epi} \]

\[ A \to B \text{ is of fp, then flat epi = ho epi} \]

\[ \text{Spec}(B) \text{ Spec} \]

\[ \text{zar open} \]

\[ A \to A_p \]

\[ P \to K \text{ epi, flat proj} \]
Notation \( R \) a Banach ring

\[ R \in \text{Ban} \quad R_{r} e \text{Ban} \]

\[ \|a\| = r \|a\| \]

rel
\[
\text{all} = r \parallel \text{all} \\
\text{re} \in \mathbb{R} \\
S_r = S_{\text{sym}}(V) \\
V = R_{r_1} \oplus R_{r_2} \oplus \cdots \oplus R_{r_n}
\]

\[
\text{Hom}(w, V) = w \otimes V \\
\text{SLin}(\text{Ban}_P) \text{ has model structure} \\
X \to K \text{ is a we/fib} \Leftrightarrow \text{AP projecty} \\
\text{Hom}(P_X, -) \to \text{Hom}(P_Y, -) \text{ is a we/fib} \\
\text{gives a model cal str}
\]
$$\forall x \in \mathbb{R}$$

$$S_r = \text{Sym}((V) \leq \mathbb{R}[x_1, \ldots, x_n]$$

$$V = \mathbb{R}_{r_1} \oplus \mathbb{R}_{r_2} \oplus \cdots \oplus \mathbb{R}_{r_n}$$

$$\text{Ind (Bamp)} \text{ has model structure}$$

$$X \rightarrow K. \text{ is a we/fib} \Leftrightarrow A$$

$$\text{Hom (P, X)} \rightarrow \text{Hom (P, Y)} \text{ is a we/fib}$$

gives a model cat str
\[ \text{all} = r \| \text{all} \]
\[ r \in \mathbb{R} \]
\[ S_r := \text{Sym} \leq (V) \leq \mathbb{R}[[x_1, \ldots, x_n]] \]
\[ V = \mathbb{R}_{r_1} \oplus \mathbb{R}_{r_2} \oplus \cdots \oplus \mathbb{R}_{r_n} \]

\[ \text{Ind}(\text{Bmap}) \text{ has model structure} \]
\[ X \to K \text{ is a we/fib} \iff \]
\[ \text{Hom}(P, X) \to \text{Hom}(P, Y) \text{ is a we/fib} \]

\[ \text{gives a model cat str} \]
\[ R_r \in \text{Ban}_R \quad a \in \mathbb{R} \quad \| a \| = r \| a \| \quad \text{in} \quad \text{Ban}_R \]
\[
S_r := \sum \{ V \} \subseteq \mathbb{R}[x_1, \ldots, x_n] \\
V = \mathbb{R} \quad \Theta \quad \mathbb{R}_n \quad \| V \|
\]

\[ S_r = \text{functions convergence on} \quad \mathbb{R}^n \quad \| c \| = r \]

\[
\text{project} \quad (Y) \quad \text{is} \quad w^* / Y^* 
\]
\[ S_n = \text{Sym}(V) \]

\[ V = R_{c_1} \oplus R_{c_2} \oplus \cdots \oplus R_{c_n} \]

Affine Algebra

\[ \frac{S_n}{I} \]

\[ f \text{ is ideal} \]
\[ V = \mathbb{R}_{c_1} \oplus \mathbb{R}_{c_2} \oplus \cdots \oplus \mathbb{R}_{c_n} \]

Affine Algebra

\[ S_r = \mathcal{R} \frac{x_1}{x_2}, \ldots, \frac{x_n}{x_2} \]

\[ S_r = R \frac{x_1}{x_2}, \ldots, \frac{x_n}{x_2} \]

\[ S_r^+ = \]
$S_r = \text{Sym}(V)$

$V = R_{e_1} \oplus R_{e_2} \oplus \cdots \oplus R_{e_n}$

$S_r = \mathbb{R} \otimes \mathbb{R} \otimes \cdots \otimes \mathbb{R}$

Affiliated Algebra

$S^+_r = \text{collin}(\text{Ind}(\text{Ban}))$

$S_r / I$ is closed ideal
\[ S_r = \text{Sym}(V) \]

\[ V = R_{r_1} \oplus R_{r_2} \oplus \cdots \oplus R_{r_n} \]

Affine Algebra

\[ S_r / I \]

\[ S_r^+ := \text{colim} \quad S_{r+1} \]

\[ \text{colim} \quad \text{Ind}(\mathbb{R}) \]

\[ \text{closed} \]

\[ \text{ideal} \]

\[ \text{fg} \]
\[ V = R_{c_1} \oplus R_{c_2} \oplus \cdots \oplus R_{c_n} \]

Affine Algebra

\[ S_r = \mathbb{R} \oplus x_{\frac{1}{n}} \oplus x_{\frac{2}{n}} \oplus \cdots \oplus x_{\frac{n}{n}} \]

Closed ideal

\[ S_r/I \]

\[ A = S_r/I \quad A^{nm} = A \]

\[ S_r \rightarrow S_{r_2} \text{ is nuclear} \]

\[ r \rightarrow r_2 \]
\[ S_r = \text{Sym}(V) \]

\[ V = R_c \oplus R_{c+1} \oplus \cdots \oplus R_n \]

Affine Algebra

\[ S_r / I \]

Closed Ideal

\[ S_r^+ := \text{colim} S_{r+n} \]

\[ c \in \text{Incl}(\text{Ban}_R) \]

\[ A = S_r^+ / I \]

\[ \chi_{uv} = A \]

Dagger Affine Algebra
$S_r = \text{Sym}(V)$

$V = R_c \oplus R_c \oplus \cdots \oplus R_c$

Affinoid Algebra

$S_r = R \{ x_1, \ldots, x_n \}$

$S_r^+ := \text{colim} S_{r^+}$

$A = S_r^+ / I$ \quad $A_{\mathbb{B}} = A$

For Affinoid Algebra

$S_r \rightarrow S_{r_2}$ is nuclear

$\eta \geq \rho$

two full cats of $\text{Comm}(\text{Ind}(\text{Ban}_R))$
Introduce certain $A \rightarrow B$, $A, B$ affinsid
$B = A^\dagger$. dagger affinsid
Introduce certain $A \rightarrow B$. $A, B$ affinoid.

Examples:

1. Laurent $A_\nu = \frac{A \sum \nu s}{(y^f - 1)}$, $f \in A$
Introduce certain $A \to B$, $A$, $B$ affine.

Dagger affine

Examples

Laurent $A_v = \frac{A^{x^2}}{\leq 3} (yf-1) \quad f \in A$

$V \subseteq M(A)$
Introduce certain $A \rightarrow B$, $A, B$ affind, dagger affind.

Examples:
- Laurent: $A^\vee = A \{ z \in \mathbb{C} : \text{Im}(zf^{-1}) \neq 0 \}$ for $f \in A$

$V \subseteq M(A)$

$\exists \epsilon f(x) \geq \epsilon$
Examples

\[ B = A_v \]

\[ A_v = \frac{3 \times 2}{yf-1} \quad f \in A \]

\[ \forall \in M(A) \]

\[ A \rightarrow A_v \]

\[ \{ x \mid |f(x)| = \frac{2}{3} \} \]

Closed
Example

1. Laurent

\[ A_{r} = A \left\{ \frac{\sum x^{2}}{5} \right\} (y^2 - 1) \quad f \in A \]

\[ V \subseteq M(A) \]

\[ A \subseteq A_{r} \]

\[ \exists \eta \mid |f(\eta)| \leq \frac{1}{5} \]

2. Weierstrass

\[ A_{r} = A \left\{ \frac{x^2}{5} \right\} (y^2 - 1) \quad g \in A \]

\[ \exists \xi \mid |g(\xi)| \leq \frac{1}{5} \]
Example:

1. Laurent
   \[ A_v = A \frac{3x^2}{x^3} \frac{1}{(y+1)} \quad f \in A \]
   \[ V \subseteq M(A) \]
   \[ A \rightarrow A_v \] (closed)
   \[ \forall x, |f(x)| < \frac{1}{3} \]

2. Weierstrass
   \[ A_v = A (3 \sqrt{3} / (1 - g)) \quad g \in A \]
   \[ \forall x, |g(x)| \leq \frac{3}{5} \]
   \[ A_c \rightarrow A_v \] (dense)
\[ f : W \rightarrow \text{cok}(\ker(f) \rightarrow V) = \ker(W \rightarrow \text{cok}f) \]

may not hold. \( f \) is strict if it holds.

\[ \text{Der } A \quad \text{Stein algebra} \]
\[ f: W \xrightarrow{\ker} V \xrightarrow{\ker} cok f \]
\[ \text{may not hold, is strict if it holds} \]

\[ \text{Def: A Stein algebra in } \text{Comm}(\text{Ind}(\text{Ban})) \]
\[ A = \lim(\ldots \to A_3 \to A_2 \to A_1) \]
Define a Stein algebra in Comm (Ind (Ban)) as a filtered limit.

\[ A = \lim_{\rightarrow} A_i \rightarrow A_{i+1} \]

Each \( A_{i+1} \to A_i \) is a strict homomorphism.
Define a Stein algebra in \( \text{Comm}(\text{Ind}(\mathbb{R})) \)

is a filtered limit

\[
A = \lim\limits_{\leftarrow} A_i := A_1 \rightarrow A_2 \rightarrow A_3 \rightarrow \ldots
\]

- Each \( A_{i+1} \rightarrow A_i \) is metastable

\[
M(A_i) \leq \text{ih}\{M(A_{i+1})\}
\]
Des $A$ Stein algebra in $\text{Comm Ind}(\text{Ban}_I)$

is a filtered limit

$A = \lim\limits_{\longrightarrow} A_3 \rightarrow A_2 \rightarrow A_i$

- each $A_{i+1} \rightarrow A_i$ is weak

$M(A_i) \leq \text{ih} + M(A_{i+1})$

Reflexive and nuclear
A = \lim (\ldots \rightarrow A_3 \rightarrow A_2 \rightarrow A_1)

- each \( A_{i+1} \rightarrow A_i \) is inclusions
- \( M(A_i) \leq \text{ih} + M(A_{i+1}) \)

Example: \( \mathbb{R}^{\aleph_3} \) is dual to

\[
\lim_{S = \frac{1}{r} - \frac{1}{n}} R_{\frac{1}{3}}^{\frac{1}{3}}
\]

- closed dist. of \( \frac{1}{r} \)
- open radius \( \frac{1}{n} \) dist

\( \Theta(A'_R) = \)
\[ \text{OCA}_R^n = \lim_{n \to \infty} R \frac{x_1}{x_1} \ldots \frac{x_n}{x_n} \]

and verification

\[ X = \text{limit of } A' \]

\[ R[x] \quad \text{OCA}'_R \]

proven homomorphism to
\[ O(CAR) = \lim_{n \to \infty} R \left( \frac{x}{n}, \ldots, \frac{x}{n}, \ldots \right) \]

and verify it.

\[ X = \text{limit of} \quad A \]

Sum \( R \) in \( \text{Im}(R) \)

Prove homotopy epi \( \Rightarrow \)

\( L(R \vee \Theta(A) \vee \text{vanish} \Rightarrow \text{proj} \)
\[ O(A_R^n) = \lim_{r \to \infty} R \sum_{x} \frac{x}{x} \Rightarrow R[A] \]

\[ \text{and verification} \]

\[ X = \text{limit of} \ A_R \]

\[ \text{Sum } R \text{ in } \text{Im}(B) \]

\[ \text{proven homotopy} \Rightarrow L \text{ Ray/ } O(A_R) \]

\[ \text{vanish} \]

\[ \text{top} \]

\[ \text{left proj} \]
Projects
1) 21Krennitzer Shaded weak C top on aff = formal ?

Dagger Affinoid Algebra
Projects

1) w/ Krenniter: shared weak C-top on affinoids = formal Zar.
2) w/ Bamboszi: same but for Dagger Situation.
2) w/ Bambozzi - same but for Dagger Situation
3) w/ Bambozzi, Kronize, Stein Story

\[ S_r \rightarrow S_{r2} \text{ is nucleon} \]
\[ r > r_2 \]
3) w/ Barotti, Kroman, Stein, Story
   recover C-analytic topology
   \( Q_0 \)

4) w/ K. Arzakov

Final Alg.

\[
\begin{align*}
\text{Ind}(\text{Ban}_R) & \quad \text{closed ideal} \\
\text{Comm}(\text{Ind}(\text{Ban}_R)) & \quad \text{Colin} \quad \text{Sr}^+ \\
\end{align*}
\]

\[
\begin{align*}
\text{Sr}_1 \rightarrow \text{Sr}_2 \text{ is nuclear} \\
\end{align*}
\]
3) w/ Bonetto, Kronert Stein Story

recover C-analytic topology

4) w/ K. Ardakov $\mathcal{J} = \text{End}_R(O, O)$ start of indec ends

$R = \mathbb{Q}_p$

$S_r = \text{"colim" } S_{r+\frac{1}{n}}$

$A = S_r^{\dagger}$ with $\text{Ind}_1^{R_n}(\mathbb{A}_n)$

Dagger Affinoid Algebra

$S_r \rightarrow S_{r_2}$ is nuclear

$r \geq r_2$
Dagger Situation

3) B. Barbieri, K. Korwia, Stein Story
Recall C-analytic topology

4) K. Ardakov
$\mathcal{J} = \text{End}_R(\mathcal{O}, \mathcal{O})$ is a sheaf of ideals

$R = \mathbb{Q}_p$ agrees with $\mathcal{J}$

$S_r := \text{colim} S_{r+\pi}$

$A = S_r^+ / I$

$A_{\pi^\infty} = A$

Dagger Affine Algebra

$S_{r_1} \to S_{r_2}$ is nuclear

$r_1 > r_2$