Abstract: I will explain how to axiomatize the notion of a chiral WZW model using the formalism of VOAs (vertex operator algebras). This class of models is in almost bijective correspondence with pairs (G,k), where G is a connected (not necessarily simply connected) Lie group and k in $H^4(BG,\mathbb{Z})$ is a degree four cohomology class subject to a certain positivity condition. To my surprise, I have found a couple extra models which satisfy all the defining properties of chiral WZW models, but which don't come from pairs (G,k) as above. The simplest such model is the simple current extension of the affine VOA $E_8\times E_8$ at level $(2,2)$ by the group $\mathbb{Z}/2$. Thus from a quantum perspective, the Lie group $E_8 \times E_8$ behaves as if it had a non-trivial center.

It is interesting to note that, unlike Chern–Simons theories whose gauge group can be disconnected, the gauge group of a chiral WZW model is necessarily connected. This raises the question of what is the chiral WZW model associated to a disconnected gauge group? Whatever the answer turns out to be, mathematicians will probably need to enlarge the class of objects that they agree to call “chiral conformal field theories” in order to accommodate these yet to be defined models.

I will present some speculations on the matter.