Title: Experimental implementation of quantum-coherent mixtures of causal relations

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Abstract: Understanding the causal influences that hold among the parts of a system is critical both to explaining that system's natural behaviour and to controlling it through targeted interventions. In a quantum world, understanding causal relations is equally important, but the set of possibilities is far richer. The two basic ways in which a pair of time-ordered quantum systems may be causally related are by a cause-effect mechanism or by a common cause acting on both. Here, we show that it is possible to have a coherent mixture of these two possibilities. We realize such a nonclassical causal relation in a quantum optics experiment and derive a set of criteria for witnessing the coherence based on a quantum version of Berkson's paradox. (Joint work with Katja Ried and Kevin Resch)
Experimental implementation of quantum-coherent mixtures of causal relations

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<table>
<thead>
<tr>
<th></th>
<th>drug</th>
<th>no drug</th>
</tr>
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<tbody>
<tr>
<td>recovery</td>
<td>60%</td>
<td>48%</td>
</tr>
<tr>
<td>no recovery</td>
<td>40%</td>
<td>52%</td>
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</table>
Cause-effect

Common cause
Pauli X

Outcomes perfectly correlated
A quantum-coherent mixture of causal relations
J. P. Maclean, K. Ried, RWS, K. Resch
arXiv:1606.04523 (quant-ph)
Can the two causal relations be mixed coherently?
Observational probing scheme

Interventional probing scheme
CLASSICAL

\[ P(CB|D) \]
Distinguishing probabilistic mixtures of common-cause and cause-effect relations

From physical mixtures of common-cause and cause-effect relations
Berkson’s paradox

Whether to date

Personality

Looks

Delightful

Acceptable

No Way

PERSONALITY

Jerk

Not so much

Could be a model

LOOKS

Pirma: 16090056

Page 16/45
Berkson’s Paradox as a diagnostic tool

- faculty position
  - teaching ability
  - research ability
Berkson’s Paradox as a diagnostic tool

Physical mixture

Probabilistic mixture

Among candidates

Among faculty

Teaching
Research

Teaching
Research

= 1/2 + 1/2
Interventionist probing $\rightarrow$ C and D are marginally independent

But they can become dependent after conditioning on B
Probabilistic Mixture of cause-effect & common-cause

There is a hidden "classical switch variable" J acting on B
For every value of J,
either
B depends only on D
or
B depends only on the common cause with C

Physical Mixture of cause-effect & common-cause

$\mathcal{E}_{CB|D}$ is not a probabilistic mixture

That is, sometimes, B depends nontrivially on both D and the common cause with C
\[ \varepsilon_{CB|D} = \sum_j P(j) \varepsilon_{BC|D}^{(j)} \]

\[ j \in \mathcal{J}_1 \]

\[ \sum_{j \in \mathcal{J}_1} P(j) \varepsilon_{B|D}^{(j)} \otimes \rho_C \]

\[ \propto \varepsilon_{B|D} \otimes \rho_C \]

\[ \sum_{j \in \mathcal{J}_2} P(j) \rho_{BC}^{(j)} \otimes \text{Tr}_D \]

\[ \propto \rho_{CB} \otimes \text{Tr}_D \]
Probabilistic Mixture of cause-effect & common-cause

\[ \mathcal{E}_{CB|D} = w\mathcal{E}_{B|D} \otimes \rho_C + (1 - w)\rho_{BC} \otimes \text{Tr}_D \]

where \( \text{Tr}_B \rho_{CB} = \rho_C \)

Physical Mixture of cause-effect & common-cause

\[ \mathcal{E}_{CB|D} \text{ is not a probabilistic mixture} \]
Each causal mechanism on its own should be quantum
Purely cause-effect

Coherent mixture of cause-effect and common-cause

Purely common-cause

\[ U = e^{i\phi} \frac{1}{\sqrt{2}} (I + iS) \]
Purely cause-effect

\[ \Phi^+ \]

Purely common-cause

\[ \Phi^+ \]

Coherent mixture of cause-effect and common-cause

\[ U = e^{i\phi} \frac{1}{\sqrt{2}} (I + iS) \]

\[ \tau_{CBD} = \frac{1}{2} (\frac{1}{2} I_C) \otimes |\Phi^+\rangle\langle \Phi^+|_{B|D} + \frac{1}{2} |\Phi^+\rangle\langle \Phi^+|_{CB} \otimes I_D \]

\[ + i \left\{ \left[ \frac{1}{2} I_C \otimes |\Phi^+\rangle\langle \Phi^+|_{B|D} \right] |\Phi^+\rangle\langle \Phi^+|_{CB} \\
- |\Phi^+\rangle\langle \Phi^+|_{CB} \left[ \frac{1}{2} I_C \otimes |\Phi^+\rangle\langle \Phi^+|_{B|D} \right] \right\}, \]
\[ U(\theta) = \cos(\theta/2)I + i \sin(\theta/2)S \]

\[ U(\theta) = \Pi^+ + e^{i\theta} \Pi^- \]

**Cause-effect:**
\[ \theta = 0 \]

**Common-cause:**
\[ \theta = \pi \]

**Coherent mixture:**
\[ \theta = \pi/2 \]
\[ U(\theta) = \cos(\theta/2)I + i\sin(\theta/2)S \]
\[ U(\theta) = \Pi^+ + e^{i\theta}\Pi^- \]
MacLean et al. “Quantum-Coherent Mixtures of Causal Relations.” *arXiv:1606.04523*
REALIZING DIFFERENT CAUSAL STRUCTURES

\[ \mathcal{E}_{BF|DE}(\rho_{DE}) = \frac{1}{2} \Pi^+ \rho_{DE} \Pi^+ + \frac{1}{2} \Pi^- \rho_{DE} \Pi^- \]
CAUSAL TOMOGRAPHY

\[ P(cbd|stu) = \text{Tr} \left[ \tau_{CBD} \cdot \Pi_{C}^{s,c} \otimes \Pi_{B}^{u,b} \otimes T_{D} \left( \Pi_{D}^{t,d} \right) \right] \]

CAUSAL TOMOGRAPHY: CHOI ISOMORPHISM

\[ \rho_B = \mathcal{E}_{B|A}(\rho_A) \]

\[ \tau_{BA} \equiv (\mathcal{E}_{B|A'} \otimes I_A)(|\Phi^+\rangle \langle \Phi^+|_{A|A'}) \]

\[ \rho_B = \text{Tr}_A \left[ \tau_{BA'} (1_B \otimes \rho_A^{T_A}) \right] \]
Common-cause process: \( U = S \)

Fidelity: 92.6%

Common-cause process: $U = S$

Fidelity: 92.6%

Cause-effect: \( U = I \)

Coherent mixture: \( U = \frac{1}{\sqrt{2}} I + i \frac{1}{\sqrt{2}} S \)

Common-cause: \( U = S \)

Fidelity: 93.7%

DIFFERENTIATING PROBABILISTIC FROM PHYSICAL MIXTURES

Covariance

\[ \text{Cov}(c, b) \equiv \langle cb \rangle - \langle c \rangle \langle b \rangle \]

Witness

\[ C_{CD} = \sum_b b P(b)^2 \text{Cov}(cd|b) = \text{Tr} \left[ \tau_{CBD} \cdot \sigma_C^s \otimes \sigma_D^t \otimes \sigma_B^u \right] \]

\[ \Rightarrow C_{CD} = 0 \]

**DIFFERENTIATING PROBABILISTIC FROM PHYSICAL MIXTURES**

**Covariance**

\[ \text{Cov}(c, b) \equiv \langle cb \rangle - \langle c \rangle \langle b \rangle \]

**Witness**

\[ C_{CD} = \sum_b b P(b) \text{Cov}(cd|b) = \text{Tr} \left[ \tau_{CBD} \cdot \sigma^x \otimes \sigma^y \otimes \sigma^z_B \right] \]

\[ \Rightarrow C_{CD} = 0 \]

SUMMARY

- New classes of causal structures
- One experimental setup can realize them all
- Reconstructed causal maps for each
- Four witnesses to classify the causal structure
- Experimentally realized a coherent mixture of cause-effect and common-cause relations.

CONCLUDING REMARKS

Where to go from here?

• Larger systems/higher dimensions
• Are there scenarios involving a physical mixture of common-cause and cause-effect where the Berkson induced correlations are not just entangled, but violate a Bell inequality? No for qubits.
• Is there a more principled way of defining coherent mixture of causal relations?
• Are there applications to information processing?