Abstract: In models of inflation driven by an axion-like pseudoscalar field, the inflaton, $a$, may couple to the standard model hypercharge gauge field via a Chern-Simons-type interaction, $L \sim a F F \bar{f}$. This coupling results in the explosive production of hypermagnetic fields during inflation, which has two interesting consequences: (1) The primordial hypermagnetic field is maximally helical. It is therefore capable of sourcing the generation of nonzero baryon number around the electroweak phase transition (via the chiral anomaly in the standard model). (2) The gauge field production during inflation feeds back into the stochastic background of gravitational waves (GWs). In this talk, I am going to discuss the correlation between these two phenomena. To this end, I will (a) present an updated study of baryogenesis via hypermagnetic fields after pseudoscalar inflation and (b) describe the corresponding implications for GWs. As it turns out, successful baryogenesis is feasible---provided the axion coupling to the gauge fields is suppressed by a decay constant $\Lambda \sim 3 \times 10^{17}$ GeV. Moreover, in the case of successful baryogenesis, one expects a characteristic peak in the GW spectrum at frequencies in the MHz range.
Magnetic Fields, Baryon Asymmetry, and Gravitational Waves from Pseudoscalar Inflation.

Kai Schmitz
Postdoc in the Particle and Astroparticle Physics Division at Max-Planck-Institut für Kernphysik (MPIK), Heidelberg, Germany

Based on arXiv:1707.07943 [hep-ph]. In collaboration with
- Daniel Jiménez: M. Sc. student at MPIK Heidelberg, Germany
- Kohei Kamada: Postdoc at Arizona State University, Tempe, USA
- Xun-Jie Xu: Postdoc at MPIK Heidelberg, Germany

Cosmology and Gravitation Seminar
Perimeter Institute | Waterloo, Canada | August 10, 2017
Take-Home Messages

Inflation driven by a pseudoscalar field (axion), coupled to the hypercharge gauge field, ...

\[ \mathcal{L}_{\text{eff}} \supset \frac{a}{4\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} \]

... entails an extremely rich phenomenology!

1. Explosive gauge field production during inflation \( \rightarrow \) primordial magnetogenesis.

2. Maximally helical hypermagnetic field \( \rightarrow \) baryogenesis via the chiral anomaly.

3. Gauge field production feeds into tensor spectrum \( \rightarrow \) source of stochastic GWs.

---

Goal of this talk: Study the compatibility / correlation of these three phenomena.
Outline

1. Pseudoscalar inflation
2. Primordial magnetogenesis
3. Baryon asymmetry of the universe
4. Signature in gravitational waves
5. Conclusions
Inflation as a pillar of modern cosmology

Inflation: a stage of accelerated expansion in the early universe

- Explains the size, homogeneity, and isotropy of our Universe on cosmological scales.
- Quantum fluctuations during inflation seed structure formation on galactic scales.

However, plethora of models in the literature:

1. No consensus on how to correctly embed inflation into particle physics.
2. How to test specific models apart from their predictions for the CMB power...
Gravitational waves and primordial black holes

Era of gravitational wave (GW) astronomy! New observational window on the early Universe.

- Primordial GWs from inflation
- Primordial black holes (→ dark matter)
Pseudoscalar inflation coupled to gauge fields

This talk: Inflation driven by a pseudoscalar field / axion-like particle / ALP / axion $a$

[Freese, Frieman, Olinto '90] [Adams, Bond, Freese, Frieman, Olinto '93]

- **Field theory:** Pseudo-Nambu-Goldstone boson of a spontaneously broken global symmetry $G_{\text{global}}$
- **String theory:** Scalar modes with a shift symmetry after compactifying the internal space

- Naturally flat potential protected by a shift symmetry $\rightarrow$ large field range
- Anomalies of global symmetry $\rightarrow$ coupling to gauge fields:

  \[ \mathcal{A} \left[ G_{\text{global}} - G_{\text{local}}^2 \right] \neq 0 \quad \Rightarrow \quad \mathcal{L}_{\text{eff}} \supset \frac{a}{4\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} \]

**Rich phenomenology:** Primordial magnetic fields, baryon asymmetry, stochastic
Primordial (hyper)magnetic fields

Our analysis: Couple inflaton to the gauge field of the standard model hypercharge $U(1)_Y$

- Minimal scenario: Abelian rather than non-Abelian gauge field; $U(1)_Y$ part of the SM.
- Gauge field production during inflation $\rightarrow$ opportunity for primordial magnetogenesis.

Primordial gauge fields may act as seed for the intergalactic magnetic fields (IGMFs) in galaxy clusters.

Primordial hypermagnetic field $B_Y$

\[ \downarrow \]

Electroweak phase transition

\[ \downarrow \]

Present-day magnetic field $B_{EM}$

- Can be probed by $\gamma$ rays from blazars. Indications for $B^0_p \gtrsim 10^{-17} \cdots 10^{-12}$

[Takahashi et al. '13] [Chen, Buckley, Ferer '15] [Finke et al. '15]
Anomalous inflation coupling to gauge fields

Lagrangian:

\[ -\mathcal{L} = \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + V(a) + \frac{a}{4\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} \]

Friedmann equation:

\[ H^2 = \frac{1}{3M_{Pl}^2} \left[ \frac{1}{2} \dot{a}^2 + V(a) + \frac{1}{2} \langle E^2 \rangle + \frac{1}{2} \langle B^2 \rangle \right] \]

Equations of motion:

\[ \ddot{a} + 3H \dot{a} + \frac{dV}{da} = \frac{1}{\Lambda} \langle EB \rangle, \quad \Box A = -\frac{a'}{\Lambda} \nabla \times A \]

- Axion-gauge-field coupling results in new source terms.
- Gauge field modes of one helicity (+ or -) are exponentially amplified.

[Turner & Widrow '88] [Garretson, Field, Carroll '92] [Anber & Sorbo '06, '10] [Durrer, Hollenstein, Jain '11] [Barnaby & Peloso '11] [Sorbo '11]
[Barnaby, Namba, Peloso '11] [Barnaby, Pajer, Peloso '12] [Meerburg & Pajer '13] [Linde, Mooij, Pajer '13]
Gauge field production during inflation

Equations of motion for the vector field modes:

\[
\frac{\partial^2}{\partial \tau^2} + \omega_k^2(\tau, \xi) A_{\pm}^k(\tau) = 0, \quad \omega_k^2(\tau, \xi) = k^2 \left[ 1 - \frac{(\pm 2\xi)}{(-k\tau)} \right]
\]

Tachyonic instability, \( \omega_k^2 < 0 \), depending on the value of the instability parameter \( \xi \):

\[
\xi = \frac{1}{2H} \frac{\dot{a}}{\Lambda}
\]

For constant \( \xi \), the modes equations are solved exactly by Whittaker \( W \) functions:

\[
A_{\pm}^k(\tau) = \frac{1}{\sqrt{2k}} \exp \left[ \pm \frac{\pi \xi}{2} \right] W_{\pm i\xi, 1/2}(2ik\tau)
\]

Physical strength and correlation length of the hypermagnetic \( B \) field at the end of inflation:

\[
B = \frac{1}{R^2} \nabla \times A, \quad B_p = \left\langle B^2 \right\rangle^{1/2} \sim 10^{-2} \frac{e^{2\xi}}{\xi^{5/2}} H^2, \quad \lambda_p = \left\langle \lambda \right\rangle \sim \cdot
\]

[Turner & Widrow '88] [Garretson, Field, Carroll '92] [Anber & Sorbo '06, '10] [Dumer, Hollenstein, Jain '11] [Barnaby & Peloso '11] [Sorbo '11]
[Barnaby, Namba, Peloso '11] [Barnaby, Pajer, Peloso '12] [Meirelles & Pajer '13] [Linde, Mooij, Pajer '13]
Gauge field production during inflation

Equations of motion for the vector field modes:

\[
\left[ \frac{\partial^2}{\partial \tau^2} + \omega_k^2(\tau, \xi) \right] A^k_{\pm}(\tau) = 0, \quad \omega_k^2(\tau, \xi) = k^2 \left[ 1 - \frac{(\pm 2\xi)}{(-k\tau)} \right]
\]

Tachyonic instability, \( \omega_k^2 < 0 \), depending on the value of the instability parameter \( \xi \):

\[
\xi = \frac{1}{2H} \frac{\dot{a}}{\Lambda}
\]

For constant \( \xi \), the modes equations are solved exactly by Whittaker \( W \) functions:

\[
A^k_{\pm}(\tau) = \frac{1}{\sqrt{2k}} \exp \left[ \pm \frac{\pi \xi}{2} \right] W_{\pm i\xi, 1/2}(2ik\tau)
\]

Physical strength and correlation length of the hypermagnetic \( B \) field at the end of inflation:

\[
B = \frac{1}{R^2} \nabla \times A, \quad B_p = \left\langle B^2 \right\rangle^{1/2} \sim 10^{-2} \frac{e^{2\xi}}{\xi^{5/2}} H^2, \quad \lambda_p = \left\langle \lambda \right\rangle \sim \frac{\xi}{H}
\]
Gauge field evolution after inflation

**Our analysis:** Instant reheating approximation + simple scaling laws after inflation. Better treatment would require dedicated numerical magnetohydrodynamics (MHD) simulation.

---

Adiabatic dilution at high temperature:

\[ B_p \propto R^{-2}, \quad \lambda_p \propto R \]

Inverse cascade below critical \( T_{ic} \):

\[ B_p \propto R^{-7/3}, \quad \lambda_p \propto R^{5/3} \]

**Inverse cascade:** Alfvén waves generate plasma turbulence on scales of size \( \lambda_T \).

Once \( \lambda_T \sim \lambda_p \), \( \lambda_p \) continues to scale like \( \lambda_T \). Transfer of energy from small to large scales.

[Bardeen & Jedamzik '04] [Brandenburg & Subramanian '05] [Kandus, Kunze, Tsaag '11] [Widrow, Ryu, Schleicher, Subramanian, Tsaag, Takaishi '14] [Kahniaev, Tovzadze, Brandenburg, Neronov '13] [Durrer & Neronov '13]
Present-day magnetic field

Physical strength and correlation length of the hypermagnetic $B$ field in the present epoch:

$$B_p^0 \simeq 3 \times 10^{-19} \text{G} \left( \frac{e^{2 \pi \zeta}}{\zeta^4} \right)^{1/3} \left( \frac{H}{10^{13} \text{GeV}} \right)^{1/2}, \quad \lambda_p^0 \simeq \frac{1.0 \text{pc}}{(4\pi)^{1/2}} \left( \frac{B_p^0}{10^{-14} \text{G}} \right)$$

Our result:

- Simple estimate. But, completely model-independent! No assumptions about $V(a)$, neglect dynamics of RH.

Compare with experimental bounds:

- CMB anisotropies, ionisation, etc.:
  
  $[\text{PLANCK '15}]$
  
  $B_p^0 \lesssim 10^{-9} \text{G}$

- Indications from blazars:
  
  $[\text{Takahashi et al. '13}, \text{Chen, Buckley, Ferrer '15}]
  
  $B_p^0 \gtrsim 10^{-17} \ldots 10^{-14}$
Baryogenesis via decaying hypermagnetic helicity

- Hypermagnetic field generated during inflation is maximally helical
  \[ \mathcal{H}_Y = \int_V d^3 x \mathbf{A} \cdot \mathbf{B} = \frac{1}{R^3} \int_V d^3 x \int \frac{d^3 k}{(2\pi)^3} k \left( |A_+^k|^2 - |A_-^k|^2 \right), \quad |A_+^k| \gg |A_-^k| \]

- Opportunity for baryogenesis via the chiral triangle anomaly in the standard model
  \[ \Delta B = \Delta L = N_g \left( \Delta N_W^{CS} - \frac{g_Y^2}{16\pi^2} \Delta \mathcal{H}_Y \right) \]
Baryogenesis via decaying hypermagnetic helicity

- Hypermagnetic field generated during inflation is maximally helical
  \[ \mathcal{H}_Y = \frac{1}{R^3} \int_V d^3 x \mathbf{A} \cdot \mathbf{B} = \frac{1}{(2\pi)^3} \int_V d^3 x \int \frac{d^3 k}{(2\pi)^3} k \left( |A_+^k|^2 - |A_-^k|^2 \right), \quad |A_+^k| \gg |A_-^k| \]

- Opportunity for baryogenesis via the chiral triangle anomaly in the standard model
  \[ \Delta B = \Delta L = N_g \left( \Delta N_W^{CS} - \frac{g_Y^2}{16\pi^2} \Delta \mathcal{H}_Y \right) \]

Competition between

- \( B+L \) production from \( \Delta \mathcal{H}_Y \neq 0 \)
  \[ \frac{d \mathcal{H}_Y}{dt} \sim \frac{2}{\sigma} \langle B \cdot \nabla \times B \rangle \]

- \( B+L \) washout by EW sphalerons
  \[ \Gamma_{\text{sph}} \approx 18 \alpha_W^5 \]
Evolution during the electroweak crossover

Source term during the electroweak crossover:

\[ S = \frac{H}{8\pi^2 s} \frac{\mathcal{H}_Y}{V} \left[ -\partial_T \theta_W(T) \right] \sin[2\theta_W(T)] \]

Efficiency controlled by temperature dependence of the weak mixing angle:

- **Z-γ mixing is proxy for B_γ to B_{em}**
- **Use phenomenological ansatz**
  \[ \cos^2 \theta_W = c_0^2 + \frac{1 - c_0^2}{2} \left[ 1 + \tanh \frac{r_0 - r}{x_0} \right] \]

- **Discrepancy between analytical calculation and lattice simulation.**
  - [Kajantie, Laine, Riumpi, Slepovichkiv '98]
  - [D'Onofrio & Rummukainen '16]

**Notes:**

- \( T_{\text{top}} \): Temperature at top
- \( T_{\text{ sợ}} \): Temperature at center
- \( T_{\text{in}} \): Temperature at inside
- \( T_{\text{out}} \): Temperature at outside

**Additional Details:**

- **Baryon Asymmetry:**
  - \( \eta_b = 10^{-16} \text{ G} \)
  - \( \omega_0 = 10^{-8} \text{ pc} \)

**References:**

- [Kamada & Long '16]
Final baryon asymmetry

- Solve complicated system of kinetic equations (incl. SM Yukawa interactions, etc.).
- Numerical result well reproduced by approximate fit formula:

$$\eta_B \simeq \frac{17}{37} \left[ g_W^2 + g_Y^2 \right] \frac{S}{\gamma_{w, sph}} ; \quad \gamma_{w, sph} \simeq \exp \left[ -147.7 + 107.9 \left( \frac{T}{130 \text{GeV}} \right) \right]$$

- Observed baryon asymmetry, \( \eta_B^{\text{obs}} \sim 10^{-10} \), reproduced for

$$B_p^{cw} \sim (0.1 \text{GeV})^2$$

or equivalently

$$B_p^0 \sim 10^{-16}$$

- How to obtain such a field strength from magnetar
Baryogenesis after pseudoscalar inflation

Our analysis of primordial magnetogenesis + BAU calculation by [Kamada & Long '16]

\[ \eta_B \simeq (6.5 \times 10^{-3} \ldots 3.8) \times 10^{-17} \left( \frac{e^{2\pi \xi}}{\xi^4} \right) \left( \frac{H}{10^{13} \text{GeV}} \right)^{3/2} \]

Prediction:

- Successful baryogenesis (mostly) based on standard model physics!
  \[ \eta_B \sim 10^{-10} \iff B^0_p \sim 10^{-16} \text{G} \]

- IGMFs have positive helicity. Testable in future (blazar halo) observations!

Largest uncertainties:

- Effect of reheating on the pseudoscalar gauge fields, exact behavior
  [Fujita et al. '15] [Adshead et al. '16]
GW production during inflation

Gauge field perturbations source tensor perturbations in the metric:

$$\left[ \frac{\partial^2}{\partial \tau^2} - \frac{2}{\tau} \frac{\partial}{\partial \tau} + k^2 \right] h_{\pm}(\tau, k) = \frac{2}{M_{Pl}^2} \Pi_{\pm}^{ij}(k) T_{ij}(\tau, k)$$

$$T_{ij}(\tau, k) = - R^2(\tau) \int \frac{d^3q}{(2\pi)^{3/2}} \left[ E_i(\tau, q) E_j(\tau, k - q) + B_i(\tau, q) B_j(\tau, k - q) \right]$$

- $h_{\pm}$: Polarization eigenstates of the transverse-traceless tensor perturbations
- $\Pi_{\pm}^{ij}$: Polarization tensor; $T_{ij}$: energy-momentum tensor induced by the gauge fields

Stochastic spectrum of chiral gravitational waves: $\mathcal{O}(H^4)$ term amplified by $e^{4\pi \xi}$. 

$$\Omega_{GW}^0 h^2 \sim \frac{\Omega_{rad}^0 h^2}{12\pi^2} \left( \frac{g_*}{g_0} \right) \left( \frac{g_{*, S}}{g_{*, 0}} \right)^{4/3} \left( \frac{H}{M_{Pl}} \right)^2 \left[ 1 + \left( \frac{H}{M_{Pl}} \right)^2 (f_L(\xi) + f_R(\xi)) \right]$$

- Numerical fit functions: $f_L(\xi) \sim 10^{-7}/\xi^6$ and $f_R(\xi) \sim 10^{-9}/\xi^6$. 

Pirsa: 17080057
Expected GW signal strength

Successful baryogenesis requires
\[
\zeta = \frac{1}{2H \Lambda} \sim 5
\]

Slow-roll inflation ends once
\[
\varepsilon \sim \frac{\dot{a}^2}{2H^2 M_{\text{Pl}}^2} \sim \frac{2\zeta^2 \Lambda^2}{M_{\text{Pl}}^2} \sim 1
\]

This fixes the suppression scale \( \Lambda \)
\[
\Lambda \sim \frac{M_{\text{Pl}}}{\sqrt{2\zeta}} \sim 3 \times 10^{17} \text{ GeV}
\]

- Inflaton must be weakly coupled. Otherwise, overproduction of BAU.
- Weak field regime: Gauge field production never dominates inflationary dynamics.
- Upper bound on GW signal strength: \( \Omega_{\text{GW}}^0 h^2 \lesssim 10^{-14} \)
Peak in the spectrum of primordial GWs

Inflationary trajectories in the $\xi - H$ parameter plane

- $\xi \propto \dot{a}/H$ increases towards the end of inflation $\rightarrow$ feature in the GW spectrum!
- Frequency determined by $H$ at the end of inflation: $f_{\text{peak}} \simeq 71 \text{ MHz } (H/10^{13} \text{ GeV})^{1/2}$
Take-Home Messages

Pseudoscalar (axion) inflation coupled to the standard model hypercharge gauge sector

\[ \mathcal{L}_{\text{eff}} \supset \frac{a}{4\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} \]

Rich phenomenology:

- Explosive gauge field production during inflation \( \rightarrow \) primordial magnetogenesis.
- Maximally helical hypermagnetic field \( \rightarrow \) baryogenesis via the chiral anomaly.
- Gauge field production feeds into tensor spectrum \( \rightarrow \) source of stochastic GWs.

Our main results:

- Baryogenesis is feasible for a weakly coupled pseudoscalar inflaton field

\[ \eta_B \sim 10^{-10} \quad \leftrightarrow \quad B_p^0 \sim 10^{-16} \text{G} \quad \leftrightarrow \quad \Lambda \sim 3 \times 10^{17} \text{GeV} \]

- Peak in GW spectrum at MHz frequencies. Out of reach of present-day tech, but in principle, smoking-gun signal for baryogenesis via decaying helicity.