Title: What is a No-Boundary Quantum State?

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Abstract: Contemporary final theories consist of a theory of the universe’s dynamics (I) like the avatars of string theory together with a theory of its quantum state (I’) like the no-boundary wave function of the universe (NBWF). This talk is concerned with the definition of a no-boundary quantum state at the semiclassical level where its predictions can be straightforwardly derived and compared with observation. A semiclassical no-boundary wave function is defined by an ensemble regular saddle points. The ensemble is restricted by simple considerations of symmetry such as time neutrality. We will briefly review the successful predictions of the NBWF so defined. We then address the question of moving beyond the semiclassical approximation by defining the NBWF in terms of a Euclidean or Lorentzian integral or by a connection with a dual field theory.
Quantum Gravity: Experiment and Tests?

It seems unlikely that we will have laboratory experiments that test quantum gravity in the immediate future.

\[ E_{\text{pl}} \equiv \sqrt{\hbar c^5/G} \sim 10^{19}\text{Gev} \]

But in the beginning and expansion of the universe we have and experiment already done where Planck energies are reached, and there is 14 Gyr of data scattered over 42 lyr of space.

Quantum Cosmology
A Quantum Universe

The results of any experiment depend on how it was set up. In cosmology that is summarized by the quantum state.

A theory of the quantum state is the objective of Quantum Cosmology.
Contemporary Final Theories Have Two Parts

Both are needed to make any predictions at all.

An unfinished task of unification?
Configuration Space

Restrict to spatially closed cosmologies. And for simplicity to a single scalar field $\phi$ coupled to the metric by Einstein gravity.

States are specified by wave functions on the configuration space of three metrics and spatial field configurations on a spacelike surface $\Sigma$.

$$\Psi = \Psi[h_{ij}(x), \chi(x)]$$
Constraints

\[ \Psi = \Psi[h_{ij}(x), \chi(x)] \]

The wave function has to satisfy the four constraints of general relativity the most important of which is the Wheeler-DeWitt equation (no matter).

\[
\left[ -\ell^2 \nabla_x^2 + \ell^{-2} h^{\frac{1}{2}} \left( -3R + 2\Lambda \right) \right] \Psi[h_{ij}] = 0,
\]

\[
\nabla_x^2 = G_{ijk\ell} \frac{\delta^2}{\delta h_{ij}(x) \delta h_{k\ell}(x)} + \text{(linear derivative terms depending on factor ordering)}
\]

\[
G_{ijk\ell} = \frac{1}{2} h^{-\frac{1}{2}} \left( h_{ik} h_{j\ell} + h_{i\ell} h_{jk} - h_{ij} h_{k\ell} \right)
\]
Semiclassical Quantum Gravity
(leading order in $\hbar$)

$$\Psi[h_{ij}, \chi] \equiv \exp\{-\hat{I}[h_{ij}(x), \chi(x)]/\hbar\}$$

To leading order in $\hbar$ the WdW equation becomes the Hamilton-Jacobi equation for $\hat{I}$ and the wave function a sum over saddle points

$$\Psi[h_{ij}, \chi] = \sum_{sp} c_{sp} \exp \{-I_{sp}[h_{ij}, \chi]/\hbar\}$$
No-Boundary Saddle Points

Extrema of the action on a 4-disk with $h_{ij}$ and $\chi$ on the boundary and are **regular everywhere inside**.
No Boundary Quantum States

= sums over no-boundary saddle points

\[ \Psi[h_{ij}, \chi] = \sum_{nbsp} c_{nbs} \exp \left\{ -I_{nbs}\left[h_{ij}, \chi\right]/\hbar \right\} \]

Natural considerations of symmetries restrict the c’s

i) time neutrality: both I and I* contribute equally.

ii) normalizability consistent with quantum mechanics.

iii) Otherwise weight saddle points equally.

Wave functions defined by different sets of saddle points can be superposed by combining the sets.
Cosmological Observations of the Universe’s Classical History

Most of our observations of the universe on cosmological scales are of properties of its classical history:

• The homogeneity and isotropy on scales above 100 Mpc. The vast age.
• The rate of expansion, the amounts of dark matter, dark energy, baryons, radiation.
• The evolution of fluctuations to make the CMB, galaxies, stars, planets, biota, us, etc.
A quantum system behaves classically when its state and Hamiltonian predict high probabilities for histories with correlations in time governed by deterministic laws.
Minisuperspace Models

Homogeneous, isotropic, and closed configurations of geometry and a scalar field.

\[ ds^2 = -dt^2 + a^2(t) d\Omega_3^2 \]
\[ \phi = \phi(t) \]

Dynamics: General Relativity, plus
\[ V(\phi) = \Lambda + \frac{1}{2} m^2 \phi^2 \]
\[ \Psi = \Psi(b, \chi). \]

No boundary wave function in the semiclassical approximation:
\[ \Psi_{NB}(b, \chi) \approx \exp \left[ -I_{\text{ext}}(b, \chi)/\hbar \right] \]

The saddle point action is generally complex
One parameter family of saddle points \( \varphi_0 \).
Predicting Quantum Probabilities for the Universe’s Classical Histories
Minisuperspace Models

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Predictions for Emergent Classical Histories

\[ \Psi(b, \chi) \approx \exp\{\left[ -I_R(b, \chi) + iS(b, \chi) \right]/\hbar \} \]

WKB: When \( S \) varies rapidly compared to \( I_R \), an ensemble of classical histories is predicted that are the integral curves of \( S \).

\[
\begin{align*}
    p_b &= \nabla_b S \\
    p_\chi &= \nabla_\chi S
\end{align*}
\]

3rd person probability of the history passing through \((b, \chi)\):

\[ p(\text{hist.}) \propto \exp[\left[ -2I_R(b, \chi)/\hbar \right]] \]

A one-parameter family labeled by \( \varphi_0 \).
Predictions for Inflation

By itself, the NBWF + classicality favor low inflation, but we are more likely to live in a universe that has undergone more inflation, because there are more places for us to be.

\[ p(\phi_0 | H_0, \rho) \propto \exp(3N)p(\phi_0) \propto \exp(3N - 2I_R) \]
<table>
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<th>Quantum Retrodictions from the NBWF</th>
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<td>classical lorentzian spacetime</td>
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The No-Boundary Quantum State of the Universe is Alive, Well, and Predictive at the Semiclassical Level.
Towards Principles for:

1) Determining the saddle point weighting in the NBWF.

2) For unifying I and $\Psi$. 
Decoherent Histories
Quantum Mechanics (DHQM)

Decoherent Histories QM ≈ Consistent Histories QM
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Quantum Mechanics (DHQM)
Decoherent Histories QM ≈ Consistent Histories QM
The most general objective of a quantum theory is the prediction of probabilities for histories.

In cosmology these are the Lorentzian histories of the universe --- cosmological histories of spacetime geometry and fields.
A Quantum Mechanics for Comology

- Fine grained histories: Lorentzian histories of geometry and field.

- Coarse Grainings: partitions of fine-grained histories into diffeomorphism invariant classes $c_\alpha$.

- A measure of interference $D$ between coarse grained histories that defines when they decohere and what their probabilities $p(\alpha)$ are.

$$D(\alpha, \alpha') = \delta_{\alpha, \alpha'} p(\alpha)$$

The construction involves an inner product and the saddle point actions have to be regular enough (normalizable) that its defined. (See J. Halliwell talk)
Beyond the Semiclassical Approximation
Integrals

There is no principle requiring a quantum state to have an integral representation, and most don’t!

\[ \Psi[h_{ij}, \chi] = \int_C \delta g \delta \phi \exp \left\{ -\frac{I[g, \phi]}{\hbar} \right\} \]

Different prescriptions for different \( C \).

Advantages:

Evaluated by steepest descent, or Picard-Lefshetz gives a prescription for which saddle points contribute with what weight in the semiclassical approximation

But the superposition of wave functions defined by integrals is generally not an integral.
Integrals are a Step Toward Unification.

\[ I \quad \Psi \]

An unfinished task of unification?

\[ \Psi [h_{ij}, \chi] = \int_C \delta g \delta \phi \exp \{-I[g, \phi]/\hbar\} \]
Duality

\[ \Psi_{SC}[h_{ij}, \chi] = \Psi[b, \tilde{h}, \chi] = \]

\[ \frac{1}{Z_{QFT}[\tilde{h}, \alpha, \epsilon]} \exp(iS_{ct}[b, \chi, \tilde{h}]/\hbar). \]

The three metric has been written as \( h_{ij} = b^2 \tilde{h}_{ij} \) where \( \tilde{h}_{ij} \) is trace-free. ZQFT is the partition function of a certain dual field theory with sources. \( \epsilon \sim 1/Hb \) is the UV cutoff in that. The sources \( (\tilde{h}, \alpha) \) of ZQFT are related to the rest of the arguments of the wave function.

Gives a prescription for which saddle points contribute with what weight in the semiclassical approximation

A step toward unification of \( I \) and \( \Psi \)
Two Comments
Lattice Gravity (Regge Calculus)

\[ \Psi_0(s_b) = \int_C d\mu(s_i) \exp[-I(s_b, s_i)] \]

\[ l^2 I = -2 \sum_{\sigma \in \partial M} A(\sigma) \psi(\sigma) - 2 \sum_{\sigma \in \mathrm{int}(M)} A(\sigma) \theta(\sigma) + 2\Lambda \sum_{\tau \in \mathrm{int}(M)} V_4(\tau). \]

- Curvature concentrated on vertices in 2d, on triangles in 4d
- A superspace of dimension of the number of edges N. No symmetry assumed.
- Path integrals over geometry become multiple integrals over edge lengths. Contours C in N complex squared edge lengths.

Sorkin
The Action For Different Topologies

4-sphere

CP2

Dot = Saddle Point
Simplicial Wave Functions

\[ \xi = \frac{s_i}{s_b} \]
Linearized Gravity (Deparametrized)

Deparametrized: Constraints solved, gauge imposed, leaves two transverse-traceless components of the perturbed metric as the true physical degrees of freedom.

Ground state: can be expressed as a euclidean functional integral but not a lorentzian one.

\[ \psi_0 [h_{ij}^{TT}, T] = \int \delta h_{ij}^{TT} \exp(-i_2 [h_{ij}^{TT}]) \]

\[ l^2 i_2 = \frac{1}{4} \int d^4 x [(\dot{h}_{ij}^{TT})^2 + (\nabla_i h_{jk}^{TT})^2] \]
Linearized Gravity (Parametrized)

Parametrization: (Faddeev-Popov) Add integrals over redundant degrees of freedom, gauge and non-gauge into the integral using identities like:

\[ 1 = \frac{1}{\sqrt{i\pi}} \int_{-\infty}^{\infty} dx \, e^{ix^2}. \quad 1 = \int_{-\infty}^{+\infty} dx \, \delta(x) \]

Can't parametrize to the covariant Einstein action because its unbounded below (conformal factor).

\[ l^2 I_2 = \frac{1}{4} \int_M d^4x \left[ (\nabla_\alpha \tilde{h}_{\beta\gamma}) (\nabla^\alpha h^{\beta\gamma}) - 2 (\nabla^\alpha \tilde{h}_{\alpha\beta})^2 \right] \]

but can reach one with the conformal factor rotated.

All orders in perturbation theory. (Kristin Schleich)

The Conformal Rotation is Not a Problem
Main Points Again

- NBWF’s are alive, well, and predictive at the level of semiclassical gravity. That is enough to compare predictions with observation.

- There are several possibly different approaches to going beyond the semiclassical approximation, integrals, dualities, etc.

- We should compete these in explaining the results of our big experiment starting at the beginning of the universe.