Abstract: Ultralight bosons can induce superradiant instabilities in spinning black holes, tapping their rotational energy to trigger the growth of a bosonic condensate. In this talk I will give an overview on superradiance and its applications focusing on the observational imprints of this process around spinning black holes which include: (i) the emission of monochromatic gravitational waves emitted by bosonic condensate formed through superradiant instabilities, potentially observable by current and future gravitational-wave detectors, and (ii) the formation of gaps in the spin versus mass plane of astrophysical black holes.
Gravitational wave searches for ultralight bosons

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Mostly based on:
In collaboration with S. Ghosh, E. Barausse, E. Berti, V. Cardoso, I. Dvorkin, A. Klein & P. Pani

Ultralight bosons?

- Black-hole superradiance can fill an important gap in the parameter space
- Roughly independent on the coupling to SM particles (CAUTION: if couplings or self-interactions are large enough, phenomenology will be different)

From: Cardoso et al '18, JCAP1803 043
Superradiance

Zel’dovich, ’71; Misner ’72; Press and Teukolsky , ’72-74

\[ \Phi(t, r, \theta, \phi) = \Psi(r)e^{-i\omega t + im\phi} P_l(\cos \theta) \]

\[ \frac{\omega}{m} < \Omega_H \]

Superradiant scattering of \textit{bosonic} waves \hspace{2cm} \Rightarrow \hspace{2cm} \text{Extraction of energy and angular momentum from the black hole}
Superradiance in the lab

\[ R = \sqrt{\frac{J_{\text{out}}}{J_{\text{in}}}} \]

Recently observed in the lab for an analogue BH.
Superradiance: amplification factors

Press and Teukolsky, ’72-74

- Amplification factors larger for spin-2 fields (gravitational waves)
- Amplification factors grow with the black hole spin
- Nonlinear effects slightly decrease the efficiency East, Ramazanoglu & Pretorius, ’14
Superradiant instability

Confinement + Superradiance \rightarrow \text{Superradiant instability}

Press & Teukolsky, '72

The Yukawa potential of a **massive bosonic field** naturally confines low-frequency waves.

**Spinning BHs are unstable against massive bosons.**

Damour '76; Zouros & Eardley '79, Detweiler '80; Dolan '07,…

Mostly relevant when:

\[
\frac{M m_b}{M_{Pl}^2} \sim \left( \frac{M}{10M\odot} \right) \left( \frac{m_b c^2}{10^{-11}\text{eV}} \right) \sim \mathcal{O}(1)
\]
Superradiant instability: unified picture

Zouros & Eardley ’79; Detweiler ’80; Dolan ’07; Pani et al ’12; RB, Cardoso & Pani ’13; Baryakhtar, Lasenby & Teo ’17; East ’17; Cardoso et al ’18; Frolov, Krtous, Kubiznák & Santos ’18,...

- **s=0**, \( \Box \Phi - \mu_S^2 \Phi = 0 \)

- **s=1**, \[
\begin{align*}
\Box A_\nu - R_\nu \mu A^\mu - \mu_V^2 A_\nu &= 0, \\
\mu_V^2 \nabla^\mu A_\mu &= 0.
\end{align*}
\]

- **s=2**, \[
\begin{align*}
\Box h_{\mu \nu} + 2R_{\alpha \beta \mu \nu} h^{\alpha \beta} - \mu_T^2 h_{\mu \nu} &= 0, \\
\mu_T^2 \nabla^\mu h_{\mu \nu} &= 0, \\
(\mu_T^2 - 2A/3) h &= 0.
\end{align*}
\]

(Note: computation assuming small-spin approximation)

\[
\delta X_{\mu_1 \ldots} (t, r, \theta, \phi) = \delta X^{(i)}_{lm}(r) \gamma_{\mu_1 \ldots}^{lm(i)}(\theta) e^{im\phi} e^{-i\omega t}
\]

\[
\delta X (r \to \infty) \propto e^{-r \sqrt{\mu^2 - \omega^2}}
\]

\[
\omega = \omega_R + i\omega_I
\]

\[
\omega_R \sim \mu
\]

\[
\omega_I \propto (m\Omega_H - \omega_R) (M \mu)^\alpha
\]
Evolution of the superradiant instability?

- During the instability phase black hole slowly loses spin and mass until it reaches saturation $\omega_R = m\Omega_H$.

- Formation of long-lived bosonic condensates around BHs (or truly stationary hairy black holes for complex fields Herdeiro & Radu ’14).

- Numerical simulations confirm linear/adiabatic predictions. East & Pretorius, ’17; P. Bosch, S. Green & L. Lehner, ’16; Sanchis-Gual et al ’16; RB, Cardoso & Pani ’15

CW sources: classical GR point of view

- Formation of “gravitational atoms” around spinning black holes:
  
  \[ \Phi = \epsilon \Re \left( \phi_{lmn}(r)S_{lm}(\theta)e^{im\varphi}e^{i\omega_R t} \right) \]

  \[ T_{\mu\nu} = -\frac{1}{4} (\Phi,_{\alpha} \Phi^{,\alpha} + \mu^2 \Phi^2) + \frac{1}{2} \Phi^{,\mu} \Phi^{,\nu} \sim \epsilon^2 \sin(2\omega_R t) \ldots \]

- These condensates are a source of very long-lived gravitational wave signals:

  \[ G^{(0)}_{\mu\nu} + \epsilon^2 \delta G^{(2)}_{\mu\nu} = \epsilon^2 \delta T_{\mu\nu} \]

- Backreaction of the scalar field onto the geometry is small (confirmed by numerical simulations).

- Gravitational-wave signal can be accurately computed within black hole perturbation theory.
Continuous gravitational wave sources

Arvanitaki, Dimopoulos, Dubovsky, Kaloper & March-Russell '09; Arvanitaki & Dubovsky, '10; Yoshino & Kodama '14; Arvanitaki, Baryakhtar & Huang, '15; ...

superradiant instability

\[ \frac{\dot{E}_{SR}}{\omega m} < \frac{\dot{J}_{SR}}{\tau_{SR}^{\text{min}}} \approx 10^7 M \]

GW emission

\[ \frac{\dot{E}_{GW}}{\omega m} = \frac{\dot{J}_{GW}}{\tau_{GW}^{\text{min}}} \approx 10^{10} M \]

f_{GW} \sim 5 \text{ mHz} \left( \frac{m_{eV}^2}{10^{-11} \text{eV}} \right),

\[ \frac{\omega}{m} = \Omega_H \]

GWS

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\[ \log_{10}(m_e/\text{eV}) \]

\[ \text{f}_{\text{Hz}} \]

\[ \text{t}_{\text{rad/(yrs)}} \]

\[ \text{t}_{\text{gw/(yrs)}} \]

\[ M_{BH} = 15M_0 \]
Gaps in the mass vs spin plane

Arvanitaki, Dimopoulos, Dubovsky, Kaloper & March-Russell ’09; Arvanitaki & Dubovsky, ’10

- Separation of scales allows for an adiabatic evolution of the system

- Observations of several BHs, with precise measurement of mass and spin, could give indications of new physics.

\[ \tau_{\text{instability}} \approx \tau_{\text{accretion}} \]

\[ \tau_{\text{accretion}} \sim 4.5 \times 10^7 \text{yr} / f_{\text{Edd}} \]

Random distributions of the initial BH mass between \( \log_{10} M_0 \in [4, 7.5] \) and \( J_0 / M_0^2 \in [0.001, 0.99] \) extracted at \( t = t_F \), where \( t_F \) is distributed on a Gaussian centered at \( \tilde{t}_F \sim 2 \times 10^9 \text{yr} \) with width \( \sigma = 0.1 \tilde{t}_F \).

RB, Cardoso, Pani, ‘14
MBH spin & mass function

- In reality MBH spin and mass evolution is very messy: semi-analytical model (Barausse 2012, arXiv:1201.5888) calibrated to data

\[ M_{bh} \text{ [}M_\odot\text{]} \]

- Low-mass end of MBH mass function mostly unconstrained:

From: Sesana, EB, Dotti & Rossi '14

From: Babak et al '17
Massive black hole binaries: merger rates

- $\text{popIII}=\text{light seeds}$
- $\text{Q3-d}=\text{heavy seeds, delays}$
- $\text{Q3-nod}=\text{heavy seeds, no delays}$

![Graphs showing merger rates vs redshift and mass.](image)

From Klein, Barausse et al '15

- Seed model: light seeds from PopIII stars ($\sim 100$ Msun) vs heavy seeds from instabilities of protogalactic disks ($\sim 10^5$ Msun)

- No delays between galaxy and BH merger, or delays depending on environment
Laser Interferometer Space Antenna

From LISA Proposal ’17, arXiv:1702.00786

“Laser Interferometer Space Antenna” (LISA) selected as ESA’s L3 Mission in 2017. Launch expected in ~2034.
Gaps in the mass vs spin plane: spin-0

- LISA can measure BH masses and spins with very good precision.
Constraining ultralight bosons

- Use Bayesian model comparison to constrain/measure mass of the boson
- LISA could rule out/detect scalar fields in the mass range $\sim [10^{-13}, 10^{-18}]$ eV
- NOTE: LISA could also detect early inspiral of stellar mass BBHs (Sesana '16)
Gaps in the mass vs spin plane: spin-1

Baryakhtar, Lasenby & Teo ’17, Cardoso et al ’18

- Exact instability timescales (i.e. valid for any spin) recently computed for massive spin-1 fields
- LISA should provide similar constraints for spin-1 fields.
- Can we measure the spin of the particle, e.g. using Bayesian model comparison?
- Similar constraints can be put on massive spin-2 fields (massive gravitons)

From: Cardoso et al ’18, JCAP1803 043

RB, Cardoso & Pani, 2013
Multi-band GW searches

Vertical lines: $a/M = 0.9; z = 0.01-3.01$ (right to left), $M \mu$ grows along vertical lines
Astrophysical models

- For LISA we use the semi-analytical models of arXiv:1201.5888 (Barausse ’12).

- For LIGO we need BH mass and spin distributions for both galactic and extragalactic stellar mass BHs. We use models described in arXiv: 1604.04288 (Dvorkin et al ’16,17) that take into account redshift-dependent SFR and metallicity. Caveat: does not predict spin distribution.

- LIGO is mostly sensitive to signals within the galaxy. However unresolvable extragalactic sources can produce a large stochastic background.
Stochastic Background from extragalactic sources

\[ \Omega_{GW} = \frac{1}{\rho_c} \frac{d \rho_{GW}}{d \ln f} = \frac{f}{\rho_c c^2} \int_{\text{SNR} < 8} dM dz d\alpha dE_s \frac{d^2 n}{dz dM d\alpha} df_s \]

\[ dE_s/df_s \approx E_{GW} \delta(f(1+z) - f_s) \]

\[ E_{GW} = \int_\Delta t dE/dt \]

\[ \Delta t = \langle \min(\tau_{GW}/(N_m + 1), t_s, t_0) \rangle \]

- The existence of many unresolved sources produces a large stochastic background with uncertainties mostly dominated by the BH spin distribution.
Back-of-the-envelope estimate

- Average mass fraction of the BH population emitted by the boson cloud is:

\[ f_{\text{ax}} \sim \mathcal{O}(1\%) \]

- The stochastic background can be estimated as:

\[ \Omega_{\text{GW, ax}} = (1/\rho_c)(d\rho_{\text{GW}}/d \ln f) \sim f_{\text{ax}}\rho_{\text{BH}}/\rho_c \]

- For BHs in the LISA band: \( \rho_{\text{BH}} \sim \mathcal{O}(10^4) M_\odot/\text{Mpc}^3 \) \( \implies \Omega_{\text{GW, ax}}^{\text{LISA}} \sim 10^{-9} \)

- For LIGO we note that for BH binaries, \( \Omega_{\text{GW, bin}} \sim f_{\text{GW}} f_{m \rho_{\text{BH}}}/\rho_c \), where \( f_{\text{GW}} \sim \mathcal{O}(1\%) \) is the binary’s mass fraction emitted in GWs and \( f_{m} \sim \mathcal{O}(1\%) \) is the fraction of stellar-mass BHs in binaries that merge in less than the Hubble time.

\[ \Omega_{\text{GW, ax}}/\Omega_{\text{GW, bin}} \sim f_{\text{ax}}/(f_{\text{GW}} f_{m}) \sim 10^2 \]

\[ \Omega_{\text{GW, bin}} \sim 10^{-9} - 10^{-8} \implies \Omega_{\text{GW, ax}}^{\text{LIGO}} \sim 10^{-7} - 10^{-6} \]
Stochastic Background from extragalactic sources

\[ \Omega_{GW} = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d \ln f} = \frac{f}{\rho_c c^2} \int_{SNR<8} d\alpha dM d\varepsilon \frac{dt}{dz} \frac{d^2n}{dMda} \frac{dE_s}{df_s} \]

\[ dE_s/df_s \approx E_{GW} \delta(f(1+z) - f_s) \]

\[ E_{GW} = \int_{\Delta t} dE/dt \]

\[ \Delta t = \langle \min(\tau_{GW}/(N_m + 1), t_S, t_0) \rangle \]

- The existence of many unresolved sources produces a **large stochastic background** with uncertainties mostly dominated by the BH spin distribution.
Resolvable sources

\[ D = \frac{\sqrt{S_h(f)}}{h_{\text{thr}}} \]

- If there is a massive light boson in the right mass range we will see it.
Conclusions

- **Measurements of the spin and mass** of pre-merger black holes detected by LISA could rule out (or detect) a large range of light bosonic fields.

- Bosonic condensates around BHs produced by superradiant instabilities can act as **continuous gravitational wave sources** which could be observed with current and future gravitational wave detectors.

- Large number of unresolved sources produces a **large stochastic gravitational wave background**.

- Join different channels and current data to start constraining parameter space.

Thank you!