Navigating the World of Quantum Matter
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- Land of Topological Order
- Isles of Schrodinger's Cats
- Sea of Quantum Criticality
- Forest of Unentangled Product States
RESEARCH AREAS INCLUDE:

- Topological phases of matter
- Critical phases of matter and exotic quantum critical points
- State-of-the-art numerical and analytic approaches to the many-body problem
- Application of modern information and complexity theory to quantum many-body physics
- Quantum error correction, CFT, and bulk locality in holography and beyond
- Non-equilibrium phenomena
- Chaos, scrambling, and complexity in quantum matter and holography

Other faculty members connected to the Initiative:

Faculty:
- Yin-Chen He
- Tim Hsieh
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- Davide Gaiotto
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Collaborators

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(Quantum) Phases of Matter

Classical: thermal fluctuations can drive a phase transition

Quantum fluctuations can also drive transition

Ferromagnet → Paramagnet

$g$ (tuning parameter in Hamiltonian)
Topological Quantum Phases

The original: Integer quantum Hall

Recent offspring: Topological materials

$\text{Bi}_2\text{Se}_3$
Fractional ones are rare

Integer
Quantum
Hall
Variety

http://www.sp.phycam.ac.uk/
Fractional ones are rare

Integer Quantum Hall Variety

Fractional Quantum Hall Variety

Challenging to realize in materials

Topological bootstrap:

TH, Y-M. Lu, and A. Ludwig, Sci. Adv. 3 (10), e1700729
Synthetic Quantum Systems

- Highly tunable system of qubits with precise characterization and control

- Superconducting circuits

- Trapped ions
Quantum Computation/Simulation

- Near term: 50-100 qubits, potentially useful for optimization problems
- Eventually: quantum algorithms like factoring
- Quantum simulation:
  - probe intractable quantum systems
Quantum Computation/ Simulation

• Near term: 50-100 qubits, potentially useful for optimization problems

• Eventually: quantum algorithms like factoring

• Quantum simulation:
  • probe intractable quantum systems
  • realization of nontrivial states of matter
Main Goal

Protocols for transforming trivial (unentangled) product state into nontrivial quantum state

- Cat state

Achieve:
- Quantum critical point
- Topological order
Main Tool

Quantum approximate optimization algorithm (QAOA)

Farhi, Goldstone, Gutmann (2014)

Original goal: find good solution to a classical optimization problem

Find string of bits satisfying as many constraints on bits as possible

In physics terms: find configuration of spins which minimizes energy, e.g. $S_1^z S_2^z$
QAOA: not adiabatic

Quantum adiabatic algorithm:

Simple Hamiltonian

\[ H_X \]

\[ |+\rangle \]

Target/cost Hamiltonian

\[ H_t \]

\[ |\psi_t\rangle \]
QAOA: not adiabatic

Quantum adiabatic algorithm:

Simple Hamiltonian

\[ H_X \]

Interpolate

Target/cost Hamiltonian

\[ H_t \]

\[ |+\rangle \]

\[ |\psi_t\rangle \]

If energy gap closes, need to go very slowly

Variational approach (QAOA)

\[
|\psi\rangle = e^{-i\beta_p H_X} e^{-i\gamma_p H_t} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_t} |+\rangle
\]

Choose evolution times to minimize energy \( \langle \psi | H_t | \psi \rangle \)
QAOA as preparation scheme for *quantum* state

**Quantum** target Hamiltonian $H_t$

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle_p = e^{-i\beta_p H_x} e^{-i\gamma_p H_I} \cdots e^{-i\beta_1 H_x} e^{-i\gamma_1 H_I} |+\rangle$$

Depth $p$ is number of operations allowed

Minimize energy cost

$$F_p(\vec{\gamma}, \vec{\beta}) = p \langle \psi(\vec{\gamma}, \vec{\beta}) | H_t | \psi(\vec{\gamma}, \vec{\beta}) \rangle_p$$


This talk: how well can QAOA prepare **non-trivial states**, which are separated from product states by a **phase transition**?
Transverse Field Ising Model

\[ H_{\text{TFIM}} = - \sum_{i=1}^{L} \delta i \delta i+1 - g \sum_{i=1}^{L} X_i \]

Paramagnet

\[ |++ + \ldots \rangle \]

\[ X |+ \rangle = |+ \rangle \]
Warmup: GHZ (Cat) State

Target: \[ |\psi_t\rangle = \frac{1}{\sqrt{2}} (|↑↑↑\ldots\rangle + |↓↓↓\ldots\rangle) \]

\[ H_t = - \sum_{i=1}^{L} Z_i Z_{i+1} \]
\[ H_X = - \sum_i X_i \]
\[ H_I = - \sum_{i=1}^{L} Z_i Z_{i+1} \]

\[ |\psi(\vec{\gamma}, \vec{\beta})\rangle_p = e^{-i\beta_p H_X} e^{-i\gamma_p H_I} \ldots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_I} |+\rangle \]

Cost function to be minimized: \[ F_p(\vec{\gamma}, \vec{\beta}) = p \langle \psi(\vec{\gamma}, \vec{\beta}) | H_t | \psi(\vec{\gamma}, \vec{\beta}) \rangle_p \]
Preparation of GHZ State

Perfect fidelity achieved at \( p = L/2 \)  

Minimum time to prepare GHZ
QAOA explores big Hilbert Space

$L = 12$, optimal GHZ preparation sequence

Entanglement growth during evolution of state
Quantum Critical State

Target: ground state of

\[
H_t = - \sum_{i=1}^{L} Z_i Z_{i+1} - \sum_{i=1}^{L} X_i
\]

\[
H_X = - \sum_i X_i
\]

\[
H_I = - \sum_{i=1}^{L} Z_i Z_{i+1}
\]

\[
|\psi(\vec{\gamma}, \vec{\beta})\rangle_p = e^{-i\beta_p H_X} e^{-i\gamma_p H_I} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_I} |+\rangle
\]

Cost function to be minimized:

\[
F_p(\vec{\gamma}, \vec{\beta}) = p \langle \psi(\vec{\gamma}, \vec{\beta}) | H_t | \psi(\vec{\gamma}, \vec{\beta}) \rangle_p
\]
Quantum Critical State Preparation

Perfect fidelity achieved at $p = L/2$

Minimum preparation time
Concrete Protocol

\[ |\psi(\vec{\gamma}, \vec{\beta})\rangle_p = e^{-i\beta_p H_X} e^{-i\gamma_p H_I} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_I} |+\rangle \]

\( L = 10, T = 5.250: \)

\( (0.2473, 0.6977, 0.4888, 0.6783, 0.5559, \)
\( 0.6567, 0.5558, 0.6029, 0.4598, 0.3068) \)

\( L = 12, T = 6.7651: \)

\( (0.2809, 0.6131, 0.6633, 0.4537, 0.8653, 0.4663, \)
\( 0.6970, 0.6829, 0.4569, 0.7990, 0.3565, 0.4304) \)

\( L = 14, T = 8.1604: \)

\( (0.3090, 0.5710, 0.6923, 0.5648, 0.5391, \)
\( 0.9684, 0.3979, 0.6852, 0.8235, \)
\( 0.4474, 0.6930, 0.6465, 0.4120, 0.4104) \)

\( L = 16, T = 9.8198: \)

\( (0.3790, 0.5622, 0.5638, 0.7101, \)
\( 0.9046, 0.3210, 0.6738, 0.8377, \)
\( 0.8616, 0.4004, 0.5624, 0.9450, \)
\( 0.5224, 0.6466, 0.4119, 0.5172) \)

\( L = 18, T = 11.1485: \)

\( (0.3830, 0.4931, 0.7099, 0.7010, 0.5330, \)
\( 0.6523, 0.6887, 1.0405, 0.3083, \)
\( 0.6215, 0.9607, 0.5977, 0.6209, \)
\( 0.5597, 0.7850, 0.5851, 0.4132, 0.4948) \)
Topologically Ordered State

\[ H_t = - \sum_{i=1}^{L} \sum_{j=1}^{L} \sigma_{i,j+1}^x \sigma_{i+1,j}^y + \sigma_{i,j+1}^y \sigma_{i+1,j}^x \sigma_{i,j}^x \sigma_{i,j}^y \]

Kitaev toric code / Wen plaquette model

Exhibit fractionalization: excitations can be neither bosons nor fermions

“Surface code” is promising for fault-tolerant quantum computer
Toric Code via QAOA

\[ H_t = - \sum_{i=1}^{L} \sum_{j=1}^{L} \sigma_{i,j+1}^x \sigma_{i+1,j+1}^y \sigma_{i+1,j}^x \sigma_{i,j}^y \]

\[ H_X = - \sum_i X_i \]

\[ H_I = - \sum_{i=1}^{L} \sum_{j=1}^{L} \sigma_{i,j+1}^x \sigma_{i+1,j+1}^y \sigma_{i+1,j}^x \sigma_{i,j}^y \]

An operator duality maps each diagonal into a TFIM chain

Diagonals can be prepared in parallel

Use optimal angles from GHZ prep

Perfect fidelity at depth \( p = L / 2 \)
Shortcuts via Long Range Interactions

Previously: achieved perfect fidelity, but with $O(L)$ circuit depth

Low-depth quantum circuits are highly desirable

Long range (decaying with power law) interactions are natural in trapped ion systems

$$H = -\sum_{i<j}^L Z_i Z_j \left( \frac{1}{|i-j|\alpha} \right) - g \sum_{i=1}^L X_i \quad 0 < \alpha < 3$$
Quantum Criticality via Long-Range Interactions

\[ H_t = -\sum_{i=1}^{L} Z_i Z_{i+1} - \sum_{i=1}^{L} X_i \]

\[ H_X = -\sum_i X_i \]

\[ H_I = -\sum_{i<j}^{L} Z_i Z_j \left( \frac{1}{|i-j|^\alpha} \right) \]

\[ |\psi(\vec{\gamma}, \vec{\beta})\rangle_p = e^{-i\beta_p H_X} e^{-i\gamma_p H_I} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_I} |+\rangle \]

Cost function to be minimized: \[ F_p(\vec{\gamma}, \vec{\beta}) = p \langle \psi(\vec{\gamma}, \vec{\beta}) | H_t | \psi(\vec{\gamma}, \vec{\beta}) \rangle_p \]
Quantum Criticality via Long-Range Interactions

L=12

Compare with short-range QAOA

Power-law interactions “compatible” with critical state?
Non-trivial quantum states via QAOA

- Explicit protocols for preparing GHZ, quantum critical, and topologically ordered states with perfect fidelity and O(L) circuit depth
- This talk: emphasis on concrete protocols
- several conceptual aspects
  - circuit complexity of quantum states
  - variational wavefunctions for numerics
To Do

- **Experiments:** carry out protocols! (with feedback)
- **Theory:**
  - Develop protocols for other interesting states
  - Understand better how QAOA approach works
  - Find a better acronym
Thanks!

Effect of Errors

Figure 8. Effect of errors of strength $\epsilon$ on the QAOA preparation of GHZ and critical states for system size $L$. Plotted is the infidelity averaged over 1000 error realizations (denoted by the overline).