Abstract: Ultralight bosons exist in various proposed extensions to the Standard Model, which can form condensates around rapidly rotating black holes through a process called superradiance. These boson clouds have many interesting observational consequences, such as the continuous emission of monochromatic gravitational waves. In this talk, I will describe the dynamics of the system when it is part of a binary black hole. I will show that the presence of a binary companion greatly enriches the evolution of the boson clouds, most remarkably through the existence of resonant transitions between growing and decaying modes of the clouds. Finally, I will sketch some phenomenological consequences, both for the gravitational waves emitted by the clouds and the finite-size effects imprinted in the waveforms of the binary signal.
Probing Ultralight Bosons with Binary Black Holes

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Work with Daniel Baumann and Rafael Porto [1804.03208]
Perimeter Institute, May 2018
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Era of Gravitational Waves

It is clear that future GW observations will transform astrophysics.

But, what can we learn about **physics beyond the Standard Model**?
New Light Particles

An interesting class of new physics consists of particles that are light and weakly-coupled to the Standard Model.

\[
\begin{align*}
\text{Spin-zero:} & \quad \begin{aligned}
\text{QCD axion} \\
\text{String axions} \\
\text{Fuzzy dark matter}
\end{aligned} \\
\text{Higher spin:} & \quad \begin{aligned}
\text{Dark radiation} \\
\text{Massive gravity}
\end{aligned}
\end{align*}
\]

Small masses are technically natural if protected by symmetries. Weak couplings imply that they could escape detection from colliders.

Essig et. al. [1311.0029]
Superradiance

**Ultralight boson condensate** can be created around a **rotating black hole**, if their Compton wavelength $\lambda_c$ is of the order of the size of the black hole.

$$\lambda_c = \frac{h}{\mu c} \lesssim \frac{GM}{c^2}$$

Zeldovich (1972)
Starobinsky (1973)
Superradiance

Stellar Mass BH  Super Massive BH  Milky Way

Size [km]

Bosons probed through black hole superradiance

Sum of neutrinos masses

\[ \mu [\text{eV}] \]

\[ 10^{-20} \quad 10^{-10} \quad 1 \]
Cloud in Isolation

The cloud is a source of continuous, monochromatic GW emission.

Arvanitaki, Dubovsky [1004.3558]
Cloud in a Binary

Focus of this talk: a binary companion introduces new scales and new dynamics.

Companion perturbs the cloud, affecting GW signal from the cloud.
Cloud perturbs the companion, affecting GW signal from the binary.
Outline

I. Cloud in isolation
   - Black hole superradiance
   - Properties of the cloud

II. Cloud in a binary
   - Quadrupole coupling
   - Resonance effect

III. New phenomenology
   - Signal from the cloud
   - Signal from the binary
I. Cloud in Isolation
Black Hole Superradiance

Wave amplification occurs when \( \frac{\omega}{m} < \Omega_H \)

where \( \omega \) and \( m \) are the frequency and azimuthal number of the wave, and \( \Omega_H \) is the angular velocity of the black hole horizon.

Zeldovich (1972)
Starobinsky (1973)
Black Hole Superradiance

A reflecting mirror surrounding the BH creates a **black hole bomb**:

Superradiance occurs until $\frac{\omega}{m} = \Omega_H$.

Press, Teukolsky (1972)
Black Hole Superradiance

**Massive fields** naturally create this reflecting mirror.
Scalar Field in Kerr Background

Scalar field of mass $\mu$ around a Kerr background:

$$(g^{ab}\nabla_a \nabla_b - \mu^2) \Psi(t, r) = 0$$

Far field limit in Boyer-Lindquist coordinates:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 - \frac{4aM \sin^2 \theta}{r} dt d\phi$$

$$+ \left(1 + \frac{2M}{r}\right) dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

where $M$ and $a$ are the BH mass and spin, respectively.
The Gravitational Atom

Define the non-relativistic field $\psi$:

$$\Psi(t, \mathbf{r}) = \frac{1}{\sqrt{2\mu}} \left[ e^{-i\mu t} \psi(t, \mathbf{r}) + e^{+i\mu t} \psi^*(t, \mathbf{r}) \right]$$

The equation of motion of $\psi$ is the Schrödinger equation with a **Coulomb-like** central potential:

$$i \frac{\partial}{\partial t} \psi(t, \mathbf{r}) = \left[ -\frac{1}{2\mu} \nabla^2 - \frac{\alpha}{r} \right] \psi(t, \mathbf{r}) + \mathcal{O} \left( \frac{1}{r^2} \right)$$

where $\alpha$ is the ‘coupling constant’

$$\alpha \equiv M \mu = \frac{\text{Gravitational radius}}{\text{Compton wavelength}}$$
The Gravitational Atom

\[ i \frac{\partial}{\partial t} \psi(t, r) = \left[ -\frac{1}{2\mu} \nabla^2 - \frac{\alpha}{r} \right] \psi(t, r) \]

As for the hydrogen atom, each eigenstate is characterised by three ‘quantum numbers’

\[ n : \text{ Principal} \]
\[ \ell : \text{ Orbital} \]
\[ m : \text{ Azimuthal} \]

The characteristic Bohr radius of the cloud is

\[ \frac{r_c}{M} \approx \frac{n^2}{\alpha^2} \]
Energy Spectrum

\[ \omega_{n\ell m}^{(0)} = \mu \left( 1 - \frac{\alpha^2}{2n^2} \right) \]  
Bohr energy

\[ \omega_{n\ell m}^{(1)} = \mu \left( -\frac{\alpha^4}{8n^4} + \frac{(2\ell - 3n + 1) \alpha^4}{n^4(\ell + 1/2)} \right) \]  
Relativistic kinetic energy  
Fine structure splitting

\[ \omega_{n\ell m}^{(2)} = \mu \left( +\frac{2(a/M)m\alpha^5}{n^3\ell(\ell + 1/2)(\ell + 1)} \right) \]  
Hyperfine splitting

Baumann, HSC, Porto [1804.03208]
Quasi-Stationary States

Unlike the hydrogen atom, these are not stationary states:

$$\omega_{n\ell m} \rightarrow \omega_{n\ell m} + i\Gamma_{n\ell m}$$

where the instability rate is

$$\Gamma_{n\ell m} \propto \frac{1}{M} (m\Omega_H - \omega_{n\ell m}) \alpha^{4\ell + 5}$$

Detweiler (1980)
Quasi-Stationary States

When superradiance saturates, only the **2p-orbital** is occupied:

\[ |n \ell m \rangle = |211\rangle \]

Since

\[ \Gamma_{211} \propto \frac{1}{M} (\Omega_H - \omega_{211}) \alpha^9 \]

which depends sensitively on \( \alpha \), the growth timescale can range from minutes to billions of years.
Radial Profile

Superradiant growth of the $|211\rangle$ mode only occurs when $\alpha < 0.5$
Stability of the Cloud

**Real** scalar fields emit continuous GW signal.

\[ P_{gw} \propto \left( \frac{M_c}{M} \right)^2 \alpha^{14} \]

which is also extremely sensitive to \( \alpha \), but suppressed compared to \( \Gamma_{211} \).

**Complex** scalar fields do not have such a signature, due to its time-independent and axisymmetric configuration.

Arvanitaki, Dubovsky [1004.3558]
Yoshino, Kodama [1312.2326]
Brito, Cardoso, Pani [1411.0686]
II. Cloud in a Binary

Baumann, HSC, Porto [1804.03208]
The gravitational perturbation by the companion can be organised by a multipole expansion.

The Newtonian potential in the **freely-falling frame** of the cloud is

$$V_* = -\frac{M_*}{R_*} \left[ 1 + \sum_{|m_*| \leq 2} \frac{4\pi}{5} \left( \frac{r}{R_*} \right)^2 Y_{2m_*}(\theta, \phi)Y_{2m_*}^*(\Theta_*, \Phi_*) + \mathcal{O}\left( \frac{r}{R_*} \right)^3 \right]$$

where

$$\{r, \theta, \phi\}$$ are the coordinates of the cloud, and

$$\{R_*, \Theta_*, \Phi_*\}$$ are the coordinates of the companion.
Time Dependence

Time dependence arises from **oscillating quadrupole**:

\[ V_* \supset \sum_{|m_*| \leq 2} - \frac{M_*}{R_*(t)} \left( \frac{r}{R_*(t)} \right)^2 Y_{2m_*}^* (\Theta_*(t), \Phi_*(t)) \propto e^{-im_* \Phi_*(t)} \]

Since the BH rotates in a preferred azimuthal direction, there are two classes of orbital orientations:

- \( \Phi_*(t) = +\Omega t \) (co-rotating)
- \( \Phi_*(t) = -\Omega t \) (counter-rotating)

Oscillating quadrupole periodically drives the gravitational atom.
Rabi Oscillations

In the hydrogen atom:

\[ f = \Delta E \]

When the frequency of the oscillating external field matches the energy difference between the two energy levels, resonant transitions occur.
Rabi Oscillations

In the gravitational atom:

\[ |m_\ast| \Omega = \Delta E \]

When the **orbital frequency** of the inspiral matches the energy difference between the two energy levels, resonant transitions occur. Unlike the hydrogen atom, the eigenstates are only quasi-stationary, and the cloud can transition to a **decaying mode**.
Level Mixings

Level mixings through the quadrupole obey certain selection rules.

\[ n = 3 \]
\[ n = 2 \]
\[ n = 1 \]
\[ \ell = 0 \quad \ell = 1 \quad \ell = 2 \]

Allowed transitions
Orientation of Orbits

The orientation of the orbit is also crucial:

\[ \langle \psi_d | V_* | 211 \rangle \propto e^{-im_*\Phi_*(t)} \propto e^{\mp im_*\Omega t} \]

\[ n = 3 \]

Counter-rotating orbits
(Bohr resonance)

\[ n = 2 \]

Co-rotating orbits
(hyperfine resonance)

\[ \ell = 1 \]

If the inspiral orbit is **counter-rotating**, resonance transition **upwards**.
If the inspiral orbit is **co-rotating**, resonance transition **downwards**.
Hyperfine and Bohr Resonances

Scalar field radial function

$M \quad M/\alpha \quad 4M/\alpha^2 \quad R_h \sim M/\alpha^2 \quad R_h \sim M/\alpha^4$

Compton wavelength  Bohr radius  Bohr resonance  Hyperfine resonance
Hyperfine and Bohr Resonances

As the orbit shrinks due to GW emission, the binary scans through the resonances.
Resonance Depletion

The mass of the cloud decays roughly as

\[
\frac{M_c(t)}{M_c(0)} \sim e^{-2A(t)}, \quad A(t) \equiv |\Gamma_d| \int^t dt' |c_d(t')|^2
\]

where $A$ is an estimator of the amount of depletion. Resonance can attenuate, $A \sim 1$, or completely deplete the cloud, $A \gg 1$. 
Resonance Depletion

Two parameters control the amount of depletion:

$$\alpha \equiv M\mu \quad \text{and} \quad q \equiv \frac{M_*}{M}$$
Resonance Depletion

System spends more time in hyperfine resonance than Bohr resonance.

\[ A(t) \equiv |\Gamma_d| \int_{t'}^t dt' \ |c_d(t')|^2 \]
Level Mixings

Level mixings through the quadrupole obey certain selection rules.
III. New Phenomenology

Baumann, HSC, Porto [1804.03208]
III. New Phenomenology

\[ M_c(t) \sim e^{-2A(t)} \]

*Time dependence* of the cloud gets imprinted on GW observables:

- Signal from the Cloud
- Signal from the Binary
Signal from the Cloud

Resonance depletion of the cloud creates a \textit{time-dependent} change in the continuous GW signal.

\[ P_{gw}(t) \propto \left( \frac{M_c(t)}{M} \right)^2 \alpha^{14} \]
Signal from the Cloud

Resonance depletion of the cloud creates a time-dependent change in the continuous GW signal.

\[ P_{gw}(t) \propto \left( \frac{M_c(t)}{M} \right)^2 \alpha^{14} \]
Signal from the Binary

*Finite-size effects* of the cloud leave imprints on the *phase* of the binary waveform:

- Spin-induced quadrupole
- Tidal deformability

Motivates precision gravity and highly accurate waveform models.

Resonant depletion of the cloud also creates a *time-dependent* change in the finite-size effects on the waveform.

Cutler et. al. [9208005]
Spin-Induced Quadrupole

Spinning motion of the cloud induces a quadrupole in the *polar* direction.

\[
\text{Energy density} \propto \sin^2 \theta
\]

Imprints on the phase of waveforms at **2PN** order.

Poisson [9709032]
Tidal Deformability

**Tidal Love number** quantifies the quadrupolar response of the cloud to the tidal force created by the companion.

Imprints on the phase of waveforms at **5PN** order.

Flanagan, Hinderer [0709.1915]
Damour, Nagar [0906.0096]
Binnington, Poisson [0906.1366]
Resonance Frequency

Resonance depletion occurs at specific GW frequency from the binary.
Summary and Outlook

Presence of an orbiting companion induces resonant transitions of the cloud to a decaying mode.

The new phenomenologies provide independent probes of the properties of the cloud, such as the mass of the scalar field.

Can we also infer other properties, such as the spins and self-interactions of the ultralight boson? Work in progress
Thank You Very Much!