Abstract: For well over a decade, we developed an entirely pictorial (and formally rigorous!) presentation of quantum theory [*]. At the present, experiments are being setup aimed at establishing the age at which children could effectively learn quantum theory in this manner. Meanwhile, the pictorial language has also been successful in the study of natural language, and very recently we have started to apply it to model cognition, where we employ GPT-alike models. We present the key ingredients of the pictorial language as well as their interpretation across disciplines.

every QF conference needs a crackpot
Video Article: The Quantum Linguist

Bob Coecke has developed a new visual language that could be used to spell out a theory of quantum gravity—and help us understand human speech.

by Sophie Hebden

Quantum Mechanical Words and Mathematical Organisms

By Joselle Kehoe | May 16, 2013 | 10
beam splitter

photons | photons

| photons | photons

| photons | photons
noise | poo
---|---
| baby |
| food | love
State :=

Test :=

Number :=
State :=

Test :=

Number :=
quantum wires and boxes
When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.

— Erwin Schrödinger, 1935.
– entanglement –
quantum teleportation

- Aleks
  error
  \[ U_i \]
  \[ \psi \]

- Bob

= 

- Aleks
  \[ U_i \]
  \[ \psi \]

- Bob

Bob's problem now!
quantum theory

any local theory
Peter Selinger (2008) Any equational statement is provable for string diagrams if and only if it is provable for Hilbert spaces and linear maps.
– completeness –

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E. Jeandel, S. Perdrix & R. Vilmart (yesterday) ... everything, even^2 better ...

Kang Feng Ng and Quanlong Wang (4 hours ago) ... everything, even^3 better ...

E. Jeandel, S. Perdrix & R. Vilmart (17.4 mins ago) ... everything, even^4 better ...

Kang Feng Ng and Quanlong Wang (3.4 secs ago) ... everything, even^5 better ...
Excited to share that the next http://unitary.fund grant has been made to to Aleks Kissinger and John van de Wetering to support pyZX https://github.com/Quantomatic/pyzx, an optimizing quantum circuit compiler based on a diagrammatic semantics. Congrats!

https://www.youtube.com/watch?v=iC-KVdB8pf0

pyZX, a new tool for quantum circuit optimisation using the ZX calculus.
Any age restrictions?
KIDS OUTPERFORM OXFORD STUDENTS AND DISCOVER QUANTUM FEATURES THAT TOOK TOP SCIENTISTS 60y
we have dictionaries for words
- Bottom part: word meanings
- Top part: grammar

Mathematics of grammar:

Lambek’s Residuated monoids (1950’s):
\[ b \leq a \circ c \iff a \cdot b \leq c \iff a \leq c \circ b \]

so in particular,
\[ a \cdot (a \circ 1) \leq 1 \leq a \circ (a \cdot 1) \]
\[ (1 \circ b) \cdot b \leq 1 \leq (1 \cdot b) \circ b \]

Lambek’s Pregroups (2000’s):
\[ a \cdot a^{-1} \leq 1 \leq a^{-1} a \cdot a \]
\[ b^{-1} \cdot b \leq 1 \leq b \cdot b^{-1} \]
Mathematics of grammar:

For noun type $n$, verb type is $-1n \cdot s \cdot n^{-1}$, so:
Mathematics of grammar:

For noun type $n$, verb type is $^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot ^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

As a diagram:
Mathematics of grammar:

For noun type $n$, verb type is $^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot ^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

As a diagram:

```
  Alice       hates       Bob
    |                   |
    v                   v
Bob       Alice
  hates

(Diagram shows a triangular structure with Alice, hates, and Bob connected)
Logical meanings:

\[ \rho_{\text{queen}} := \]

from language to cognition
Algorithm for NLP-meaning composition:

1. Perform grammatical type reduction:
   \[(\text{word type } 1) \ldots (\text{word type } n) \sim \text{sentence type}\]

2. Interpret diagrammatic type reduction as NLP-map:
   \[f :: \begin{array}{c}
   \bigcirc \\
   \bigcirc \\
   \end{array} \rightarrow \left( \sum_i \langle ii \rangle \right) \otimes \text{id} \otimes \left( \sum_i \langle ii \rangle \right)\]

3. Apply this map to tensor of word NLP-states:
   \[f \left( \overrightarrow{v}_1 \otimes \ldots \otimes \overrightarrow{v}_n \right)\]
Books by famous developmental psychologist:

books by famous developmental psychologist:


we made it compositional:

- conceptual spaces -

(a) The RGB colour cube

(b) Property $p_{\text{yellow}}$

(c) Property $p_{\text{green}}$

(a) The taste tetrahedron

(b) The property $p_{\text{sweet}}$
**convex algebra** := set $A$ and ‘mixing’ function:

$$\alpha : D(A) \to A$$

$$\alpha(|a\rangle) = a \quad \alpha\left(\sum_{i,j}p_iq_{ij}|a_{ij}\rangle\right) = \alpha\left(\sum_i p_i \left| \alpha\left(\sum_j q_{ij}|a_{ij}\rangle\right)\right)\right)$$

**convex relation** := relation that ‘commutes with mixtures’:

$$\forall i : R(a_i, b_i) \implies R\left(\alpha\left(\sum_i p_i|a_i\rangle\right), \alpha\left(\sum_i p_i|b_i\rangle\right)\right)$$

**ConvRel** := compact closed category of these
relevant structures:

- tensor := (non-cartesian) cartesian product
- cups := also like in Rel
- spiders := like ONB ones in Rel
\(- \text{naively}\)
more profoundly

banana

\[ := |\text{green}\rangle|\text{bitter}\rangle|\text{hard}\rangle + |\text{yellow}\rangle|\text{sweet}\rangle|\text{soft}\rangle \]
- more profoundly -

banana
:= |green⟩|bitter⟩|hard⟩ + |yellow⟩|sweet⟩|soft⟩

\[
\begin{align*}
green \text{ banana} &= \text{ bitter banana} \quad \text{yellow banana} = \text{ sweet banana} \\
&= \text{ hard banana} \quad \quad = \text{ soft banana} \\
&:= |green⟩|bitter⟩|hard⟩ \quad := |yellow⟩|bitter⟩|hard⟩
\end{align*}
\]
Phrase example:
MEANING ⊗ GRAMMAR (i.e. are intertwined)

- Ambiguous grammar depending on meaning.
- Order of adjectives “large red” vs. “red large”
- 3D(+1) spatial connotation of prepositions like: “on (the table)”, “next to (her)”, “after (the game)” etc.
Transitive verb sentence:

- lion
- hunts
- prey
Transitive verb sentence:

A lion understands grammar, since he is very aware of:

- action of hunting
- him/her being hunter
- wants to get a prey
Sentences $\rightarrow$ MovieClips
– visual perception –

*MovieClips* \(\rightarrow\) *Language*

Origin of grammar?
quantum → diagrams → language → cognition

