Title: Numerical Simulation of the Axion Field through the QCD Phase Transition

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Abstract: We perform a full (3+1)-dimensional numerical simulation of the axion field around the QCD epoch. Our aim is to fully resolve large dynamical non-linear effects in the inhomogenous axion field. These effects are important as they lead to large overdensities in the field at late times. Those overdensities will eventually evolve into axion minicluster, which have various phenomenological implications like microlensing events. It is therefore important to have a reliable estimate of the number of overdensities and their mass relation.
Simulations of a Cosmological Axion through the QCD Phase Transition

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Dark Matter Key Facts

• ~23% of the total energy in the Universe is matter of an unknown kind

• Many viable explanations exist: One of the best motivated is **axion dark matter**

• Much of its properties today depend on the dynamics in the early Universe
Axions

- Axions originally introduced to solve the **strong CP problem**:

\[
\mathcal{L} = \theta \frac{1}{16\pi^2} F_{\mu
u}^a \tilde{F}^{\mu
u a} \quad \longrightarrow \quad \mathcal{L}_{\text{axion}} = (\partial_\mu a)^2 + \frac{(a/f_a + \theta)}{32\pi^2} F \tilde{F}
\]

- U(1) PQ symmetry **spontaneously broken** at high scale, roughly \( f_a \approx 10^{12} \) GeV

- **Axion mass** is small (QCD effects),

\[
m_a^2 \approx \frac{m_a^2 f_\pi^2}{f_a^2}
\]

as are its couplings

\[\downarrow\]

good cold **DM candidate**

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Page 4/54
Post- vs Pre-inflationary scenario

Two different scenarios can be considered:
Breaking the PQ symmetry before or after inflation

before inflation:

\[ f_a > \frac{H_I}{2\pi} \]

\[ \rho(\theta_0) \]

\[ \Omega_{a,0} \sim \theta_0^2 \]

two free parameter: \( \theta_0 \), \( f_a \)

after inflation:

\[ f_a < \frac{H_I}{2\pi} \]

\[ \rho(\theta_0 = \pi/8) \]
\[ \rho(\theta_0 = 1) \]
\[ \rho(\theta_0 = 0.1) \]
\[ \rho(\theta_0 = 10^{-3}) \]
\[ \rho(\theta_0 = \pi/2) \]

\[ \Omega_{a,0} \sim \langle \theta_0^2 \rangle \]

one free parameter: \( f_a \)

inhomogeneous at small scales
**PQ phase transition:**

@ \( T \approx f_a \)

\[
V(\Phi, T) = \frac{\lambda}{4} (|\Phi|^2 - f_a^2)^2
\]

Formation of a string network

**QCD phase transition:**

@ \( T \approx 1 \text{ GeV} \)

\[
V(\Phi, T) = \frac{\lambda}{4} (|\Phi|^2 - f_a^2)^2 + m_a(T)^2 f_a^2 [1 - \cos \text{Arg(}\Phi\text{)}]
\]

Formation of domain walls
But what are the implications today?

Topological defects introduce additional energy and fluctuations

Structure formation!

For example: **Axion Miniclusters**

K. Zurek et al. (2006)

J. Enander et al. (2017)
But what are the implications today?

Topological defects introduce additional energy and fluctuations

\[ \downarrow \]

Structure formation!

For example: **Axion Stars**

![Axion Star Diagram](image)

NIST/JILA/CU-Boulder

Axion star radius vs mass

Rescaled radius $\tilde{R}$ [m]

Rescaled mass $\tilde{M}$ [GeV]

Visinelli et al. (2017)
Radio Signals from Neutron Stars

K. Gill
Radio Signals from Neutron Stars

\[ f = \frac{m_a}{(2\pi)} \text{ [Hz]} \]

- RX J0806.4-4123
- INS in M54
- SGR J1745-2900 (NFW)
- SGR J1745-2900 (DM spike)
- QCD Axion
- CAST
- ADMX
- ADMX Projection

A. Hook et al. (1804.03145)
Radio Signals from Neutron Stars

\[ m = 10^{-14} M_\odot, \ \delta = 1 \]

![Graph showing mass flux density over time with a peak at around 2.759 × 10^7 s]
Radio Signals from Neutron Stars

![Graph showing mass flux density over time]
Questions that needs to be answered:
How much of the total mass is in clusters?
How heavy are they?

This ultimately boils down to:
What is the spectrum of overdensities after the QCD PT?

Simulations are needed!
Getting started

- State before the QCD PT: White noise with high modes that already entered the horizon redshifted accordingly

- Problem: Creating such a initial state properly difficult (Result often depends on the details -> not good)

- Instead: Evolve a white noise initial state through the PQ PT first!
Equation of Motion for PQ breaking

Lagrangian:
\[ \mathcal{L} = \frac{1}{2} |\partial \Phi|^2 - V(\Phi, T) \]

with temperature dependent potential:
\[ V(\Phi, T) = \frac{\lambda}{4} (|\Phi|^2 - f_a^2)^2 + \frac{\lambda T^2}{6} |\Phi|^2 + m_a(T)^2 f_a^2 (1 - \cos \text{Arg}(\Phi)) \]

Decompose and reparameterise:
\[
\begin{align*}
\psi_1'' + \frac{2}{\eta} \psi_1' - \nabla^2 \psi_1 + \frac{1}{H_1^2} \left[ \lambda \psi_1 \left( \eta^2 f_a^2 (\psi_1^2 + \psi_2^2 - 1) + \frac{1}{3} T_1^2 \right) - m_a^2(T_1) \eta^{2+n} \left( \frac{\psi_2^2}{(\psi_1^2 + \psi_2^2)^{3/2}} \right) \right] &= 0 \\
\psi_2'' + \frac{2}{\eta} \psi_2' - \nabla^2 \psi_2 + \frac{1}{H_1^2} \left[ \lambda \psi_2 \left( \eta^2 f_a^2 (\psi_1^2 + \psi_2^2 - 1) + \frac{1}{3} T_1^2 \right) + m_a^2(T_1) \eta^{n+2} \left( \frac{\psi_1 \psi_2}{(\psi_1^2 + \psi_2^2)^{3/2}} \right) \right] &= 0
\end{align*}
\]

saxion: \[ \sqrt{\psi_1^2 + \psi_2^2} \]

axion: \[ \arctan \frac{\psi_2}{\psi_1} \]
Constructing an initial state

In the early Universe:
The two fields are in a \textit{thermal equilibrium}

Relevant quantity: \( \omega_k = \sqrt{m_{eff}^2 + k^2} \)

We find:
\[
\frac{m_{eff}^2}{a_1^2 H_1^2} \approx \frac{\lambda}{3} \left( \frac{T}{T_0} \right)^2 \left( \frac{T_1}{f_a} \right)^2 \gg 1
\]

\[
\omega_k \approx \sqrt{\frac{\lambda}{3}} T,
\quad n_k = \frac{1}{e^{\sqrt{\lambda/3}} - 1}
\]

Only one relevant parameter (and this even changes only the overall amplitude irrelevant after PQ phase transition once radial mode reaches its vev)
Unitless: 
\[ \psi_1'' + \frac{2}{\eta} \psi_1' - \nabla^2 \psi_1 + \left[ \lambda \psi_1 \left( \eta^2 (\psi_1^2 + \psi_2^2) - 1 \right) + \frac{1}{3} \left( \frac{T_1}{f_a} \right)^2 \right] = 0 \]
\[ \psi_2'' + \frac{2}{\eta} \psi_2' - \nabla^2 \psi_2 + \left[ \lambda \psi_2 \left( \eta^2 (\psi_1^2 + \psi_2^2) - 1 \right) + \frac{1}{3} \left( \frac{T_1}{f_a} \right)^2 \right] = 0 \]

with 
\[ \left( \frac{T_1}{f_a} \right)^2 = \sqrt{\frac{45}{4\pi^2 g_*}} \frac{M_{pl}}{f_a} = 237320. \times \frac{10^{12} \text{GeV}}{f_a} \]

Solve this numerically on a **3-dimensional grid**:
- box of 2040 x 2040 x 2040 = 8.5 billion grid sites
- periodic boundary conditions
- \( \text{O}(1 \text{ million}) \) time steps
- \( \text{O}(10 \text{ quadrillion}) \) function calls
- Computation time \( \text{O}(1 \text{ day}) \) on \( \text{O}(1000) \) CPUs
- 374 GB of memory needed
- Output several TB
Strings

Domain Wall

\[ m_a^2 (T_1) \eta^{2+n} \left( \frac{\psi_2}{(\psi_1 + \psi_2)^{3/2}} \right) \]

\[ m_a^2 (T_1) \eta^{n+2} \left( \frac{\psi_1 \psi_2}{(\psi_1^2 + \psi_2^2)^{3/2}} \right) \rightarrow \eta^{8.68} \sin \theta \]
\[ m_a^2(T_1) \eta^{2+n} \left( \frac{\psi_2^2}{(\psi_1^2 + \psi_2^2)^{3/2}} \right) \]

\[ m_a^2(T_1) \eta^{n+2} \left( \frac{\psi_1 \psi_2}{(\psi_1^2 + \psi_2^2)^{3/2}} \right) \]

\[ \eta^{8.68} \sin \theta \quad \text{for increasing mass} \]
mass growth truncated eventually:

\[ m_a(T_1) \eta^{2+n} \left( \frac{\psi_2}{(\psi_1^2 + \psi_2^2)^{3/2}} \right) \]

\[ m_a(T_1) \eta^{n+2} \left( \frac{\psi_1 \psi_2}{(\psi_1^2 + \psi_2^2)^{3/2}} \right) \]

\[ \eta^{8.68} \sin \theta \rightarrow \eta^2 \eta_s^{6.68} \sin \theta \]
$\eta = 8.00005$

$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$
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$\eta = 8.00005$

$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$
Oscillons carry a significant amount of the total mass:

![Graph showing mass fraction in δ > δ₀ over δ₀ range from 10^-1 to 10^5. The graph includes a green line labeled "Kolb & Tkatchev '94" and a blue line labeled "this result." The graph highlights a shoulder due to oscillons.](image-url)
These results are preliminary! There are three extrapolations to be made:

1. Simulating out to late times in $\eta$

   Problem: Computation time + numerical noise starts dominating

However: We know how the axion field and oscillons scale with time once we are sufficiently away from the QCD PT

Extrapolation can be done analytically
These results are preliminary! There are three extrapolations to be made:

2. **Extrapolating the quartic coupling** $\lambda \rightarrow 10^{57}$

   Problem: Strings/Domain Walls become too small to be resolved

   However: Final result should only depend logarithmically on this coupling (and is thus often ignored).

Possible Solution: Runs at different couplings and subsequent fit to mass-delta data can be extrapolated
These results are preliminary! There are three extrapolations to be made:

3. **Extrapolation to** $f_a \approx 10^{12}$ GeV

Problem: Small decay constant

- Large axion mass
  - Oscillation frequency increases
  - Smaller timestep size needed
  - More computation time needed
  - Mass needs to grow longer
  - Oscillons are longer stable
  - Oscillons have higher concentration
  - Not enough spacial resolution
Conclusions

• Post-inflationary scenario leads to small scale structure: Axion miniclusters, axion stars, ...

• To predict the mass that is bound in such objects simulations through the QCD phase transition needs to be done

• Oscillons play an important role for the creation of compact objects

• Preliminary results show that a significant amount of energy is stored in oscillons and thus in clusters
Thank you!