QFT II

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Conformal (Field Theory) = CFT

- CFT with additional spacetime symmetries

(theorem: conformal is largest admissible symmetry of a nontrivial CFT)

superconformal symmetry
Conformal transformations: scale transformation.
\[ \begin{align*}
\mathbf{x} & \rightarrow \lambda \mathbf{x} \\
\mathbf{t} & \rightarrow \lambda^z \mathbf{t}
\end{align*} \]
\( \lambda \in \mathbb{R} \), \( z \) : dynamical critical exponent

Relativistic CFT \( z = 1 \)
\( \mathbf{x}^\mu = (t, \mathbf{x}) \)

\( \exists \) system \( u \) \( z = 1 \)?

\( z = 2 \) in free Schrödinger equation
\[ \frac{\partial^4}{\partial^2 \mathbf{x}^2} \mathbf{\Phi} = \frac{-\nabla^2 \mathbf{\Phi}}{2} \]
conformal trans. $\Rightarrow \tilde{g}^\mu (x)$

- preserve angles
- but not necessarily length scales

$ds^2 \rightarrow \epsilon \ ds^2$

scale. $ds^2 \rightarrow \lambda \ ds^2$
why CFTs?

1. Asymptotic low energy behavior of a QFT. $\Rightarrow$ Scale invariant
   
   a. "Trivial" $\langle \alpha \rangle \propto l^0 \Rightarrow 0$ as $l \to \infty$

   $\Rightarrow$ Topological CFTs (TCFTs) $\Rightarrow$ Topological phases of matter

b. non-trivial CFT.
2. Any massive CFT is a deformation of a CFT

Free massless CFTs are conformally invariant.

\[ \Delta \phi + \lambda \phi^3 = 0 \quad (0) \quad D=4 \]

\[ \nabla \phi = 0 \quad (1) \]

\[ D_{\mu} F^{\mu\nu} = 0 \quad (2) \]

\[ D_{\mu} F^{\mu\nu} = 0 \quad (3) \]
For a CFT classically
\[ \Rightarrow x(\mu) \] energy scale
\[ \mu \frac{d}{d \mu} x(\mu) = \beta(\lambda) \]

NOT quantized mechanically
\[ T_{\mu \nu} \neq 0 \] conformal symmetry is broken
\[ \lambda(\lambda) \]
\[ \frac{d}{dx} \chi(x) = \beta(x) \]

mechanically

\[ \eta^{\mu\nu} \neq 0 \quad \text{conformal symmetry is broken} \]

\[ \phi \rightarrow \chi \beta(x) \]

\[ \Theta = \frac{1}{4} \phi^4 + \cdots \]

mass 2 3 4

\[ S = S_{\text{F}} + \int \Theta \Delta d^4x \quad \left( \frac{E}{M} \right) \rightarrow \]

\[ \text{erg Spree} \]

What happens in IR (low energies)

Depends on scaling dimension of \( \Theta \)

\( \Delta = \text{eigenvalue under dilatation of} \quad \Theta \)

1) \( \Delta > D \) \( \Theta \) irrelevant operator.

(unimportant at low \( E \))

2) \( \Delta < D \) \( \Theta \) relevant

(important at low \( E \))

3) \( \Delta = D \) (marginal)
3. CFTs induce an ordering in the space of CFTs?

UV CFT$_1$ + SL0 relevant

IR CFT$_2$

Assign a "height" function to CFTs such that induce an ordering in space of CFTs.

Cov $> \text{CIR}$

Rules out potential endpoints of RG flows.
C \rightarrow - \text{conformal anomalies}

\rightarrow Z \left[ \mathcal{S} \right]

CFTs such that

of CFTs.

\text{20 flaws.}
4. CFTs are relevant for mature, i.e. phase transitions.

3d Zising model

\[ H = -J \sum_{\sigma_j} \sigma_i \sigma_j + B \sum_{\sigma_j} \sigma_j \]

\[ \sigma_i = \pm 1? \]

Phase diagram:
- Ordered phase, \( M \neq 0 \)  
  - \( \mathbb{Z}_2 \) broken
  - \( M < 0 \)

- Disordered state, \( M = 0 \)  
  - \( \mathbb{Z}_2 \) unbroken

- Critical temperature

\( G_i \rightarrow G_i \)  
\( \mathbb{Z}_2 \) symmetry
For a phase transition:

- $T_c$: Curie temperature
- Second order phase transition
- $Z_2$ symmetry
- $\psi_0 \rightarrow \psi_1$

$\Gamma (T - T_c)$

$\Delta^3 = \frac{3\Delta^3}{4}$

Driven by thermal fluctuations
CFTs describe quantum phase transition \((T=0)\)

\[ \lambda \rightarrow \lambda^* \quad \Delta(\lambda) \rightarrow 0 \]

described by a CFT

1D Ising model
7. CFTs are central to string theory.

6. CFTs are relevant for Quantum Gravity.

AdS/CFT correspondence

7. CFTs on 1+1d tube. Consistency

CFT on the tube

CFT at boundary

HI tube
\[ \left( \frac{s^2}{s^2 + 1} - g \frac{0^2}{0^2} \right) \]

\[ g = 1 \text{ is a CFT} \]

6. CFTs are relevant for Quantum Gravity

\[ \text{AdS/CFT correspondence} \]

... to string theory.

CFT on 1+1d tube. Consistency

CFT on the tube

CFT at boundary

Quantum Gravity

- Einstein
- Yang Mills
- Dirac