L: Tu/Th 1 - 2:30
Tut: F 18, 27; M 5 2:30 / M 12, 17, 24, 31 3:00
Office: F 13, 20; 25 2:30 / M 19, 26; A2 2:30
Cosmology

Instructor: Matt Johnson
Fellow: Ghashal Geshjizani
TA: Juan Cayuso
Part I (FLL-MS)

- GR
- FRW Universes
- Distances
- Stats
- BRN
- Recombination / Dark Matter
- Inflation

Part II (M10 - A9)

- Cosmological
- Perturbation Theory
- Growth of Structure
- CMB
- Inflation (again)
Cosmology

GR Atomic

Astro Stats Stat Mech

Nuclear Particle

Quantum E+M Gravity
Goal of Cosmology: learn about history, composition, and future of our universe based on imperfect records of the past.

Can inform us about new physics:
- dark matter
- dark energy
- quantum gravity
- new particles
What do we observe?

Redshift of light (proxy for distance)

Standard candles (proxy for distance)

Standard rulers (proxy for distance)

Age of stars (proxy for distance)

Number of objects / varying intensity of emission or absorption (trace structure)

Cosmic microwave background (traces structure, proxy for distance)
Geometry + distribution of Structure
(distributions + time)

Theoretical cosmology → predict
Standard Cosmological Model

$\Lambda$CDM (I)
Standard Cosmological Model

$\Lambda$CDM (I)

- Cosmological Constant
  - (accelerated) exp.
- Cold dark Matter
  - (glue holds structure)
- Inflation
  - (initial conditions)
6 parameters

$S_2c\ 3$

$S_2b\ 3$

$H_0\ 3$

$A_3\ 3$

$\Omega_s\ 3$

Geometry - Spatially flat, homogeneous, isotropic, expanding (accelerated)

Structure - Adiabatic, Gaussian, nearly scale-invariant, initially tiny density fluctuations
6 parameters

\[ \begin{align*}
S_2c \\
S_2b \\
H_0 \\
A_s \\
\eta_s \\
Z
\end{align*} \]

- Geometry - Spatially flat, homogeneous, isotropic, expanding (accelerated)
- Structure - Adiabatic, Gaussian, nearly scale-invariant, initially tiny density fluctuations
- Astrophysics - Mostly transparent until CMB
Most basic observation

* The sky is mostly dark,
and looks same in all directions.

Assumption: Universe is homogeneous
(Same everywhere)
and isotropic
(Same from any angle)
1 Mpc

10 kpc

10 Mpc

Andromeda

galaxy Cluster

in any angle

1 pc ~ 3.2 kpc

Sun

M2 10 pc
* Scale of homogeneity \( \sim 100 \, \text{Mpc} \)

* Count all the Stars in a galaxy \( N^* \)
* Scale of homogeneity \( \sim 100 \text{ Mpc} \) 

* Count all the stars.

\[
\frac{N_{\text{galaxy}}}{(10^4 \text{ pc})^3} \sim 10^2
\]
\[ P_\times \sim 1 \, \text{M}_\odot/\text{pc}^2 \]

\[ \text{Cluster: } N_\Delta \sim \left(\frac{10 \, \text{Mpc}}{1 \, \text{Mpc}}\right)^3 \sim 10^3 \]

\[ N_x \sim 10^3 \times 10^2 \sim 10^5 \]

\[ \rho_x \sim 10^5 \, \text{M}_\odot/\left(10 \, \text{Mpc}\right)^3 \sim 10^{-6} \, \text{M}_\odot/\text{pc}^3 \]
Universe: $\bar{\rho} \sim 10^{15} \frac{M_0}{(10-100 \text{ Mpc})^3} \sim 10^{-9} - 10^{-6} \frac{M_0}{p^2}$ (\text{n. 10^{-7} M_0^2 \text{ measured}})

"Structure": $S = \frac{P_x - \bar{P}}{\bar{P}} \geq 1$

- galaxy: $S \sim 10$
- cluster: $S \sim 10$
- supercluster: $S \sim 1$
Av's Density of Stars:

\[ n = 10^{-7} / \text{pc}^3 \]

If all like Sun:

\[ R_* = R_{\odot} = 7 \times 10^{-8} \text{pc} \]

\[ L_* = L_{\odot} = 4 \times 10^{26} \text{W} \]
Night Sky

\[ P_r = n \left( \frac{\pi R_k^2 \chi}{2} \right) \]

Note that \( P_{\text{rot}} = 1 \) when

\[ \chi = \frac{1}{\pi R_k^2} \sim \left( 10^3 \text{pc}^3 \right) \left( 10^8 \text{pc}^3 \right) \sim 10^2 \text{pc} \sim 10^{17} \text{Mpc} \]
How bright would night sky be if Universe was larger than $10^{17}$ Mpc?
How bright would night sky be if Universe was larger than $10^{17}$ Mpc?

$$\Sigma = \frac{L_0}{4\pi r^2} = \frac{\pi R^2}{4\pi r^2} = \frac{\pi}{r^2}$$

$\Sigma$ = const

Ast
\[ \Rightarrow \text{Homogeneous + isotropic, cannot be infinitely big/old.} \]

**Crude estimate of age of Universe**

\[ \Sigma \text{obs} \sim 10^{-17} \frac{W}{m^2 \text{arcsec}^2} \]

\[ \Sigma \text{sun} \sim 10^{-3} \frac{W}{m^2 \text{arcsec}^2} \]

\[ 1 \text{arcmin} = \frac{1}{60} \]

\[ 1 \text{arcsec} = \frac{1 \text{arcmin}}{60} \]
\[ \frac{P_{\text{He}}}{\Sigma_{\text{obs}}} \sim 10^{-14} \sim n(\chi \pi R_{\text{He}}^2) \]

\[ \Rightarrow \chi \sim 10^{-14} \times 10^4 \text{ mpc} \sim 10^3 \text{ Mpc} \sim 1 \text{ 6pc} \]

\[ c_{+0} = 16 \text{ pc} \]

\[ t_0 \sim 3 \text{ Gyr} \]
Language of Cosmology - General Relativity

Basic objects: Tensors - objects that describe some physical property in a coordinate-independent way.
Vectors

\[ \mathbf{B} = B^m \mathbf{e}_m \]

Components

Basis vectors

Coordinate transformation:

\[ B^{\mu'} = \Lambda^{\mu'}_\nu B^{\nu} \]

\[ \mathbf{e}^{(\mu')} = \Lambda^{\mu'}_\nu \mathbf{e}^{(\nu)} \]

\[ \Lambda^{\mu'}_\nu = \frac{\partial x^{\mu'}}{\partial x^{\nu}} \]
\( \hat{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z} \)

\[ A^m = A'^m \]

\[ \rho_m = \rho'_m \]

\[ \hat{B} = B \]

\[ \gamma^m = \frac{\partial x^m}{\partial x'^n} \]
dual vectors - maps from vectors to real #’s

\[ \omega = \sum_{m} \Omega^{(m)} \hat{e}_{(m)} \]

\[ \Omega^{(m)} = \Theta^{(m)} \]

\[ \Rightarrow \omega(v) = \sum_{m} v^{m} \hat{e}_{(m)} \]

\[ = \sum_{m} v^{m} s^{m} = \sum_{m} v^{m} \]

\[ = \sum_{m} \]
(k,l) tensor - Map from k dual vectors and l vectors to real #s

\[ T = T^{u_1,...,u_k}_{v_1,...,v_l} \]

\[ \hat{e}^{u_1} - \hat{e}^{w_k} \] (v_1) ... (v_l)
Metric tensor: Most important tensor in GR

\[ g = g_{\mu\nu} \frac{dx^\mu}{dX^\mu} \rightarrow \text{components} \quad \text{basis vectors} \]

inverse metric: \[ g^{-1} \text{ by } g_{\mu\nu} g^{\nu\rho} = \delta^\rho_\mu \]
1) Inner product
\[ g(v, w) = g_{\mu \nu} v^\mu w^\nu \]

2) Norm
\[ g(v, v) = g_{\mu \nu} v^\mu v^\nu \]

3) Orthogonality
\[ g(v, w) = 0 \]
41) Maps vectors → dual vectors (lower indexed)

\[ g(v_i) = g^{\mu \nu} V_\mu \, dx^\nu \]

\[ = V_\mu \, dx^\nu \]

\[ = g^{\mu \nu} g_{\nu \lambda} \partial \]

5) Dual → vectors

\[ g^{-1} \]

\[ = \]
6) Defines invariant dist:
\[ ds^2 = g(d\mathbf{x}, d\mathbf{x}) = g_{\mu\nu} d\mathbf{x}^\mu d\mathbf{x}^\nu \]

7) Defines volume measure:
\[ V_q = \int d\mathbf{x} \sqrt{-g} \]