\[ \text{Obs}^a (H_g) = H(X, (\mathcal{N}, T_x)^a_{\text{sym}}) \]  
\[ H_g \text{ SM horizon + open g-handle} \]

Today: Quantization

Set-up: Let \( g \) be a Riemann metric on \( M \)

- \( Q_{\text{conf}} = d \zeta \)

- Heat Kernel: For \( t > 0 \), \( K_t \in \text{Sym}^2 (E) \) s.t.

  \[ \langle K_t (x, y), \phi (y) \rangle = \left( \exp (-[Q, Q_{\text{conf}}]) \phi \right) (x) \]

- Factor as \( K_t (x, y) = K_{t+} (x, y) \otimes \omega^2 \)

- Scale + BV Laplacian: \( \Delta_t \colon \text{O}^2 (E) \rightarrow \text{O}^2 (E) \) contract w/ \( K_t \)

- \( \text{effective propagator: } P^t \zeta = \int_0^t \left( Q_{\text{conf}} \otimes 1 \right) K_{t-s} \)
BU QUANTIZATION of
ROZANSKY-WITTEN MODEL II

Last time classical theory

$(X,\omega)$ hol sympl, $M$ closed 3-mfld.

fields: $E = \mathcal{X}_M \otimes \mathcal{O}_X [1]$, $g_x = \mathcal{T}_x \otimes \mathcal{O}_X [1]

pairing: \langle - , \cdot \rangle \in \mathfrak{g} \otimes \mathfrak{g}

\text{action} \quad S(\alpha) = \int_M \kappa \lambda \beta

= \int_M d\alpha M + \sum_{k \geq 1} \frac{1}{(k+1)!} \langle \eta_{k}(\alpha \otimes \theta_{k}), \alpha \rangle

\text{Obs}^\circ (H_x)

Today: Quantum

Set-up lectures
- QMF
- Heat kernel
- Scale +
- Effective
\[ L = 0 \quad \text{present \ } K \]

\[ L = 0. \]

\[ L \text{ OME} \]

\[ \prod \text{O}^{2} \text{E} \]

\[ 3 \text{LeR}_{20} \]

\[ \lim_{L \to 0} \text{I} \{ L \} = I_{u} \mod n \]

Want: \[ \text{I} \{ L \} = W(\text{P}^l, I_{u}) \]

but \[ \text{P}^l_{0} \text{ is only smooth on } (M \times M) \setminus \Delta \]

Approach: compactify \[ (M \times M) \setminus \Delta \] and argue that \[ \text{P}^l_{0} \text{ smoothly extends} \]
that satisfies the CME A quantization

Is a collection of functions \( \{ \mathcal{I}[L] + \mathcal{O}(\varepsilon)^2 \} \) for \( L \in \mathbb{R}_{>0} \)

1. \( \mathcal{I}[L,z] = W(P_{\mathcal{E}}^z, \mathcal{I}[L]) \)
2. Asymptotic locality.
3. \( \forall L > 0, \mathcal{I}[L] \) satisfies the scale L CME.

\[ Q_{\mathcal{I}} = Q_{\text{Pauli}} + \frac{P(\mathcal{E})}{\hbar} \]

\[ \mathcal{I}[L,z] = 0 \]

Approach
Compactified configuration spaces (Kontsevich-Avrutin-Singer)

Let \( V = \mathbb{E}^1 \), \( n \geq 2 \), \( S \subseteq V \) sth \( |S| \geq 2 \)

\( M^S = \mathbb{E} S \rightarrow M^3 \), \( \Delta_S \subseteq M^S \), the small diagonal.

\[ \mathcal{B}L(M^S, \Delta_S) \text{ manifold with } \mathbb{A}. \]

\[ \mathcal{B}L^m \approx \mathcal{S}(N_\Delta_S). \]

\[ \mathcal{B}L^m = M^2 \setminus \Delta_S. \]

\[ \text{Defn.: let } M^0 \text{ be the disjoint config space} \]

\[ M^0 \rightarrow M^0 \times \prod_{\mathcal{B}L(M^S, \Delta_S)} \text{ the compactified config space } \mathcal{M}_0 \]

is the closure of this embedding.
Prop - propagator $P^\mu_0$ on $(M \times M) \Delta$

can be lifted to a smooth
2-form on $M[2\mathcal{J}]$, which we denote $\tilde{P}^\mu_0$. Furthermore,

Here's an explicit formula for $\tilde{P}^\mu_0 |_{\gamma \in M[2\mathcal{J}]}$

Idea of proof - carefully analyze small $\gamma$ asymptotics of $P^\mu_0$, do

an expansion near the diagonal $\Delta$
Let $V(Y)$ denote the vertices of a graph $Y$.

$S \subseteq V(Y), \ M[V(Y)] \xrightarrow{\text{ne}} M[\bar{s}(e), t(e)3]$.

$e \in E(Y), \ M[V(Y)] \xrightarrow{\Pi_e} M[\bar{e}(e), t(e)3]$.

$v \in V(Y), \ M[V(Y)] \xrightarrow{\Pi_v} M$.

Def: Let $Y$ be a connected, stable graph not containing loops or tails, $|T(Y)| = k$.

$$W_r(P_0^L, I_0) = \sum_{M[V(Y)]} \prod_{e \in E(Y)} \Pi_e(P_0^L) \prod_{v \in V(Y)} \Pi_v.$$
Combinatorial part of graph walk

\[ w(u(l_z(e_i \otimes e_j), \phi)) \]

\[ = -w(u(l_z(e_j \otimes e_i), \phi)) \]
BU QUANTIZATION OF
ROZANSKY-WITTEN MODEL II

(Chan-Leung Li 1502.03510v3)

Defn: - Name quantization of $I_u$ is defined by

\[
I_{\text{name}} [L] = \sum_{r, \text{ conn ex}} \frac{\tau^r(L)}{|\text{Aut} r|} W_r (P_r, I_u)
\]
\[ \text{Quantum Master Eq.} \]

\[ QME \rightarrow \frac{1}{\hbar} \left( (d_m + V) \mathcal{I} \right) \cdot \exp \left( -t \mathcal{I} \right) \cdot \mathcal{I} \]

\[ = -\left( \nabla^2 \right) \cdots + Q^0 \exp \left( -t\mathcal{Q}^0 \right) + \frac{\partial \rho}{\partial t} \]

\[ \text{Hamiltonian} \]

\[ d_m (\tilde{P}_0) = -K_L \]

[Marked suffices to check in (M = M) \cdot \Delta]

\[ d_m (P) = K_0 - K_L \]

[Only simplify for \Delta]
\[
\begin{align*}
& d_{\mu} \left( W_{\gamma} \left( \tilde{P}_{\alpha}, I_{\beta} \right) \left( \phi_{1}, \ldots, \phi_{k} \right) \right) \\
& = \sum_{i=1}^{k} W_{\gamma} \left( \tilde{P}_{\alpha}, I_{\beta} \right) \left( \phi_{1}, \ldots, \phi_{i}, \ldots, \phi_{k} \right) \\
& = \int \frac{d \left( \prod_{e \in \mathcal{E}(\gamma)} P_{\alpha} \prod_{i=1}^{k} \phi_{i} \right)}{M \prod_{e \in \mathcal{E}(\gamma)} \prod_{i=1}^{k} \phi_{i} + \sum_{e \in \mathcal{E}(\gamma)} \frac{K_{e}}{M}} + \int \frac{\sum_{e \in \mathcal{E}(\gamma)} K_{e}}{M \prod_{e \in \mathcal{E}(\gamma)} \prod_{i=1}^{k} \phi_{i}} \\
& \text{need to understand boundary state of } M \left[ \nu \left( \gamma \right) \right] \quad \text{if } e_{0} \text{ is nontopological, cancels w/ th terms in } \mathcal{I} \mathcal{I} \mathcal{L}_{\alpha} \quad \text{if } e_{0} \text{ is topological, cancels w/ th terms in } \mathcal{I} \mathcal{I} \mathcal{I} \mathcal{I} \mathcal{I} \mathcal{I} \mathcal{L}_{\alpha} \text{ not involving } e_{0}.
\end{align*}
\]
$M[V]$ is a disjoint union of open strata.

$M[V] = \bigcup_{S \in S} M(S)^\circ$

$S$ is a collection of subsets of $V$ with
- $S, S_1, S_2 \in S$ either
- $S_1, S_2 = \emptyset$ or $S_1 \subseteq S_2$
- every $S \in S$, $|S| \geq 2$

**Facts:** each $M(S)^\circ$ is open

For codim 1 strata $S = \Sigma S^3$
\[ V = V_{1}, \ldots, V_{3}, \quad S = S_{1}, \ldots, S_{3} \quad 2 \leq k \leq n \]

\[ E = S(N_{S}) \xrightarrow{\pi} \Delta_{S} \]

\[ M(S)^{0} = \xi(e, x_{1}, \ldots, x_{|V_{S}| - 1}) \in E \times M^{\prod_{i \in S_{1} - 1} \pi_{i}(e), x_{1}, \ldots, x_{|V_{S}| - 1}} \]

Assume that the vertices in \( S \) are connected in \( T \)

(\( \delta_{\alpha} \) integral vanishes for type reasons)

\[ \text{Step 1: if } |S| \geq 3, \text{ then the integral as } M(S)^{0} \text{ vanishes.} \]
All de boundary sheets w/ \( S = V(Y) \) prime.

Idea: can express sum of all boundary integrals as an RA Plan of primitive boundary integrals.

Let \( Y' \subseteq Y \) be 5th

\[ V(Y') = S \]

\[ E(Y') = \text{edges in } Y \text{ incident to } S \]

\[ T(Y') = \text{half edges of } Y \text{ incident to } S \text{ but not part of internal edges.} \]
\[ W_{r,s} (\hat{P}_{o}, I_{a}) = \sum_{N(S) = \ldots} \prod_{i=1}^{k} p_{r}^{i} \prod_{i=0}^{t} \phi_{i}^{r,s} \]

\[ W_{r,s} (\hat{P}_{o}, I_{a}) = W_{r,s} (\hat{P}_{o}, I_{a}, W_{r,s} (\hat{P}_{o}, I_{a})) \]
Quantum Master Eq.

\[ QME \leftarrow \frac{1}{\hbar} (\frac{\partial}{\partial t} + \nabla) I[I]\left[ I, I[I] + \varepsilon I[I, I[3]] \right] e^{-\frac{1}{\hbar} \int_0^t dt' Q_{ac} \exp \left( t [Q, Q_{ac}] + \frac{p(R)}{\hbar} \right)} \]

Lemmma:
\[ d_m(\tilde{P}_0) = -K_L \]

Remark:
\[ d_m(\tilde{P}_0) = K_0 - K_L \]

only support on \( \Delta \)