Abstract: In this talk I will discuss some recent results on boundary degrees of freedom (or edge modes), and their description via an extended phase space structure containing extra boundary fields. Motivated by a slight modification of the covariant phase space formalism, I will show how the use of a boundary Lagrangian enables to include the edge modes in the phase space and to obtain their boundary dynamics. This will be illustrated on the example of Maxwell theory, where in addition the edge modes can be understood as contributing to entanglement entropy. The possible extension of this result to gravity will then be mentioned.
Edge modes / boundary actions / dynamics
Motivations

- Do edge modes exist? Where? How? What?
- Appear everywhere: holography
- Many approaches
- 1st order theories, CS/Maxwell
2. 1st Maxwell

- Conservation of both E and M charges → H.S. SJ
- Soft theorem for scalar
- Are there "hidden" charges?
- 1st order formulation
Edge modes / boundary actions / dynamics

\[ S = \int B \wedge F + * B \wedge B \]
\[ \Rightarrow \begin{cases} dB = 0 \\ *F = B \end{cases} \]
\[ \Rightarrow d*F = 0 \]
\[ q(\alpha) = \int_{\Sigma} \alpha \wedge B \]
\[ q(\phi) = \int_{\Sigma} \phi \wedge A \]

\[ \delta_{\alpha} A = 0 \]
\[ \delta_{\phi} B = 0 \]

\[ \phi \text{ is a 1-form} \]

\[ (\omega(\alpha), \omega(\phi)) = \int_{\Sigma} \phi \wedge d\alpha \]

First order theories / CS / Maxwell
\[ \text{On-shell: } Q(x) \to \int x \star F \]

\[ \{ \Omega, \psi \} = \int d\phi \cdot d\alpha \]

\[ \Phi = \tilde{\phi} \quad Q(\phi) \to \int d\phi \wedge A \]
1st order formulation

\[ \phi = \phi \]  

\[ \Omega(\phi) \rightarrow \int d\phi^* \wedge A \]

3d Maxwell

4d Scalar \rightarrow *F \wedge F

3-form
\[ S \cdot B = 0 \]

\[ \left( S \cdot B = d\phi \right) \quad \phi \text{ is a form} \]

\[ \{ \mathcal{Q}(\alpha), \mathcal{Q}(\phi) \} = \int S \phi \, d\alpha \]

\[ \text{3) 1st order GR} \]

\[ L[g] = \sqrt{g} R \quad \rightarrow \quad L[e, \omega] = * F \eta e F + \eta F \]

\[ \Omega_{\text{mak}}^\mu = (g^\mu_k g^{\nu}_l \nabla^k \nabla^l - g^{\mu}_k D^k \nabla^l) \delta g_{\nu}^l \]

\[ \Theta_{\text{rad}} = B \wedge \omega \approx \Theta_{\text{metric}} + d(-) \]

\[ \omega = \omega(e) \]
Metric \quad \text{diffeo-charge} \quad Q(\mathcal{G}) = \text{Koma}_1 = \int \star d\mathcal{G} = \int_{\Sigma} \epsilon_{\mu
u} \partial_{\mu} \mathcal{G}^\nu \\
\text{tetrad} \quad \text{diffeo} \quad Q(\mathcal{G}) = \int_{\Sigma} \mathcal{G} - \mathcal{B} \wedge \omega \\
\text{Lorentz} \quad \mathcal{Q}(\alpha) = \int_{\Sigma} \alpha (B^I)^I
Proposal by DePaoli, Speciale, Blau

\[ \Theta_{\text{tetrad}} = \Theta_{\text{tetrad}} + \phi(\beta e \sigma e - \epsilon \epsilon e e) \]

\[ \Rightarrow \text{then} \quad \begin{cases} Q'(y) = \text{Koma} \\ Q'(x) = 0 \end{cases} \]

\[ \theta = B \frac{\delta w}{\epsilon} \]

\[ = (\epsilon \epsilon e + \beta e e) \delta w \]
Boundary action

\[ \int_\mathcal{M} BF + \int_\mathcal{M} \mathcal{B} \text{Ind} \omega e - (\# \text{ree}) \mathcal{E} \int_\mathcal{M} dS^I S^S + \mathcal{GHV} \text{ term} \]

Variation

\[ SS = \int_\mathcal{M} \text{Eom} + \int_\mathcal{M} \text{O} + \mathcal{S} \int_\mathcal{M} \mathcal{H} \]

Boundary metric

\[ g_{ab} = M^{IJ} (a^I b^J) \rightarrow \mathfrak{sl}(2\mathbb{R}) \text{ algebra part} \]
\[ q_e = 0 - q_e \]

\[ = B_u S \omega - d(\tau e^I - 2(\tau e^I) \int_{S^I} S^I) \]

\[ Q(\tau) = \int \epsilon \tau e^I \tau e^J d w^I + \frac{1}{2} \]

\[ = \int \epsilon^I \epsilon^J \eta_{\epsilon^I \epsilon^J} P_{\alpha \beta} \text{ Brown-York charge} \]

\[ \Phi_{\tau} = \Phi_{\tau} \]

\[ V(\tau) = \int \epsilon P_{\alpha \beta} \]

\[ S = \int \epsilon P_{\alpha \beta} - \frac{1}{2} \epsilon P_{\alpha \beta} + \frac{1}{2} \epsilon P_{\alpha \beta} \]
Boundary action
\[ \delta L_{\text{H}} \]
\[ \int_B F_H + \int_{\partial M} B e \wedge \omega - \int_B \ast d \psi \leq 0 \]

GHY term

Variation
\[ \delta S = \int_M \delta \mathcal{S} + \int_{\partial M} \mathcal{S} \]
boundary potential