Title: Summer Undergrad 2020 - Path Integrals (M) - Lecture 3

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Collection: Summer Undergrad 2020 - Path Integrals

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Abstract: Perturbation theory

Lecture 3 - Perturbation Theory

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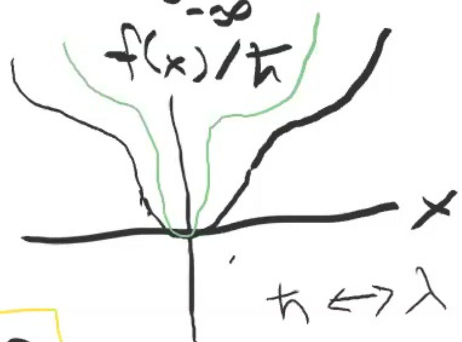
small parameter

$$S_E = \int d\tau \left(\frac{1}{2} m \dot{q}^2(\tau) + \frac{1}{2} m\omega^2 q^2(\tau) + \frac{\lambda}{4!} q^4(\tau)\right]$$

$$K_E = \int \mathcal{D}q(\tau)\, e^{-S_E[q(\tau)]/\hbar} \quad \text{vs} \quad I = \int_{-\infty}^{\infty} dx\, e^{-f(x)/\hbar}$$



$$K_E \longleftrightarrow I$$

$$q(\tau) \longleftrightarrow x$$

EOM

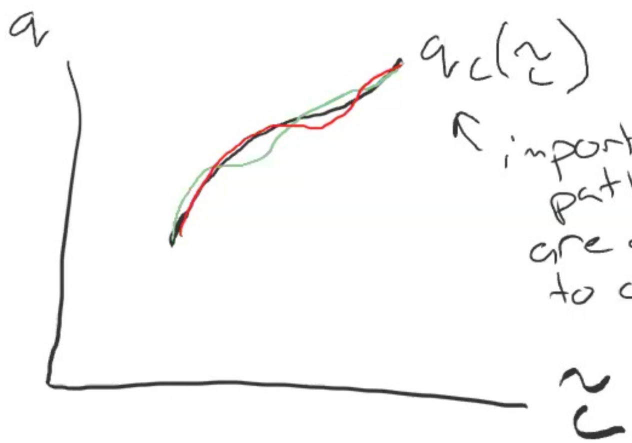
$$S_E \longleftrightarrow f$$

$$\frac{\delta S_E}{\delta q(\tau)} = 0 \longleftrightarrow \boxed{f' = 0} \qquad \boxed{f'' > 0}$$

$$\left(S_E'' > 0\right)$$

$$q_c(\tau) \longleftrightarrow x_c \quad \text{minima}$$

$$\hbar \longleftrightarrow \lambda$$



important paths are close to classical paths

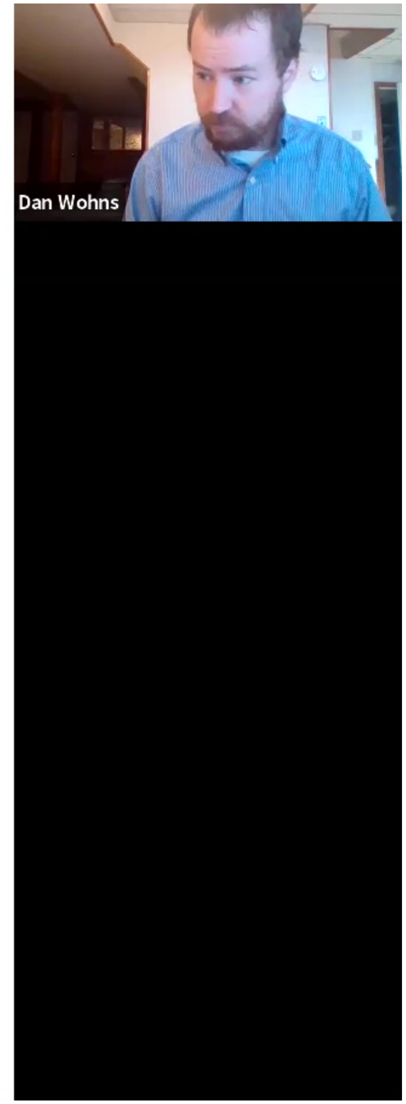
breakout rooms

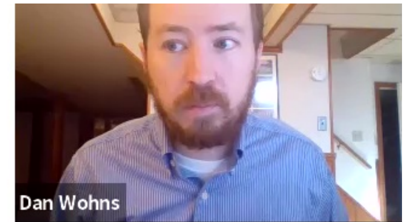
strategy: Taylor expand f

$$I = \sum_{c_1, c_2, n} c_1 \int_{-\infty}^{\infty} dx\, x^n e^{-c_2 x^2}$$

classical source J

$\int \mathcal{D}q(\tau)$





$$\int \mathcal{D}q(\tau)\exp\left[-\frac{1}{\hbar}\int d\tau\left(\frac{1}{2}m\dot{q}^2(\tau)+\frac{1}{2}mq^2(\tau)-J(\tau)q(\tau)\right)\right]$$

$$\leftrightarrow \int dx\, \exp[-x^2+xJ]$$

$$\frac{d}{dJ}\left(\int dx\,\exp[-x^2+xJ]=\sqrt{\pi}\,e^{J^2/4}\right)$$

$$\int dx\, x\exp[-x^2+xJ]\Big|_{J=0}=\sqrt{\pi}\,\frac{J}{2}e^{J^2/4}\Big|_{J=0}$$

$$\int dx\, x\exp[-x^2]=0$$

$$\exp\left[\frac{d}{dJ}\right]=1+\frac{d}{dJ}+\frac{1}{2}\frac{d^2}{dJ^2}+\cdots$$

Semiclassical limit

$$I = \int_{-\infty}^{\infty} dx\, e^{-f(x)/\hbar}$$

$$= \int_{-\infty}^{\infty} dx\, e^{-\frac{1}{\hbar}\left(f(x_c) + \frac{1}{2} f''(x_c)(x-x_c)^2 + \frac{1}{3!} f'''(x_c)(x-x_c)^3 + \dots\right)}$$

$$1 - \frac{f'''(x_c)}{3!\,\hbar}(x-x_c)^3 + \frac{f^{(4)}}{4!\,\hbar}(x-x_c)^4 + \dots$$

$$y = \frac{x - x_c}{\sqrt{\hbar}}$$

$$\simeq e^{-\frac{1}{\hbar}f(x_c)} \int_{-\infty}^{\infty} dy \sqrt{\hbar}\, e^{-\frac{1}{2} f''(x_c)\, y^2} \Big(1 - \underbrace{\frac{\hbar^{3/2}}{\hbar} y^3}_{=0 \text{ by symmetry}} - \frac{\hbar^2 y^4}{\hbar} \dots$$

$$\lambda \leftrightarrow \hbar$$

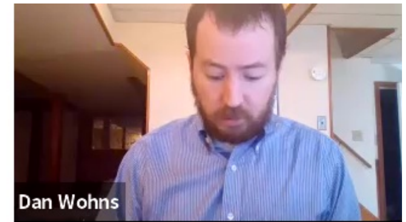
$$\int dx \exp[-x^2]\exp[-\lambda x^4] = \int dx \exp[-x^2]\sum_{j=0}^{\infty}\frac{(-\lambda x^4)^j}{j!}$$

$$\overset{?}{=} \sum_{j=0}^{\infty} \int dx \frac{(-\lambda)^j}{j!} x^{4j}\exp[-x^2]$$

$$I(\lambda) \underset{\lambda\to 0}{\sim} \sum_{j=0}^{\infty} a_j \lambda^j$$

$$\lim_{\lambda\to 0^+} \frac{I(\lambda)-\sum_{j=0}^{N} a_j\lambda^j}{\lambda^N} = 0 \quad \text{for all } N$$

For fixed small λ, usually good approximation for small N



$$\lim_{\lambda \to 0^+} \frac{e^{-\frac{1}{\lambda}}}{\lambda^N} = 0$$